Algebraic Expressions

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Starter worksheet

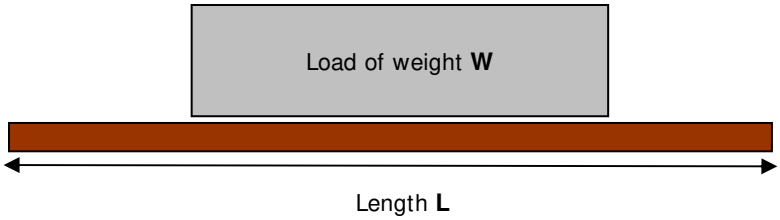
Suppose we have a wooden beam with a heavy weight on it.

As we put heavier and heavier weights on the beam, it will bend more. Eventually it may break. Suppose that you're designing a building with several storeys. You would need to decide how thick the floor needs to be. You can't just construct the building and then test it by holding a big party on the top floor – it's too dangerous.

"When we're designing a building, we imagine that there's a heavy person – like a rugby player or a super heavyweight boxer – standing on each square metre of the floor. We work out how thick the floor needs to be to hold all these people without bending by more than a few millimetres. Then we make the floor even thicker than that. There are standard legal safety margins which we work with."

- Chris Bean, Structural Engineer

How do the engineers work out how thick the floor has to be? They use some simple algebra.



For a beam of length **L** metres with a load of weight **W** kilograms sitting on it, the deflection **D** is the amount in metres that the beam bends.

Scientific testing shows that there's an equation which relates **D**, **W** and **L**:

$$D = c \times W \times L^3$$

where **c** is a constant depending on things like the type of wood used, how thick the floor is and whether the weight on the beam is all at one point or spread evenly along the beam.

Calculate the deflection **D** in each of the following cases.

1. **c** = 0.0001, **W** = 10 kg, **L** = 2 m
2. **c** = 0.0001, **W** = 50 kg, **L** = 2 m
3. **c** = 0.0001, **W** = 50 kg, **L** = 3 m
4. **c** = 0.0001, **W** = 150 kg, **L** = 3 m

What do you observe? Write down three things that you notice.

(For example, does the beam bend more with the same weight if it is longer?)

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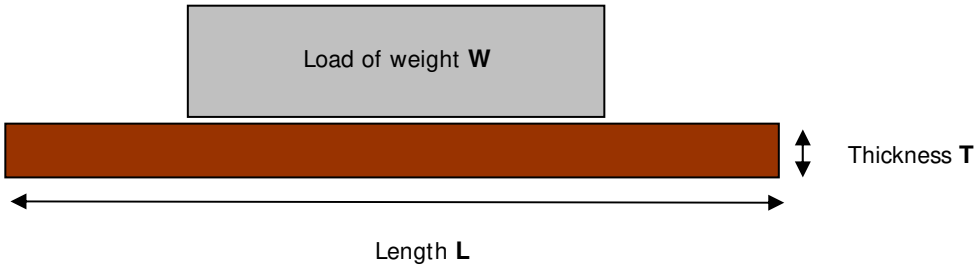
Core worksheet

For safety, beams in a building have to cope with heavy weights – just in case an international rugby team comes to visit! They also have to be the right length for the building design. This means that if a team of engineers calculate that heavy weights (or heavyweights!) will cause a beam to deflect too much, they can't just make the beam shorter. Instead, they can try to decrease the deflection by making the beam thicker.



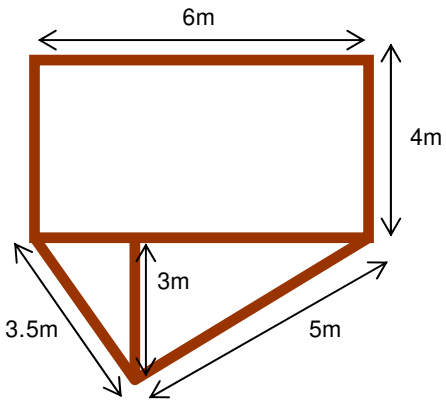
For a beam of length **L** metres and thickness **T** metres with a load of weight **W** kilograms sitting on it, the deflection **D** is the amount in metres that the beam bends. The equation we have been using to calculate the deflection is $\mathbf{D = c \times W \times L^3}$.

This is not very useful if we want to see how the thickness of the beam affects the deflection. However, the constant **c** can be calculated using another equation involving a number **k** whose value depends on the type of wood.

$$\mathbf{c = \frac{k}{900000\ T^3}}$$

Use this to write a new equation for D.

The diagram shows the structure an engineer wants to build. A beam is **safe** if it deflects no more than 0.002m when holding 1500kg.



The rectangle is built out of one type of wood, where k= 0.0001. Calculate how thick each must be to be safe:

1. the 6m beam
2. the 4m beam

The other beams are made of a different type of wood, where k= 0.0002. For each beam, work out if it is safe. If it is not safe, how thick does it need to be?

3. The 3m beam is 0.13m thick.
4. The 3.5m beam is 0.23m thick.
5. The 5m beam is 0.25m thick.

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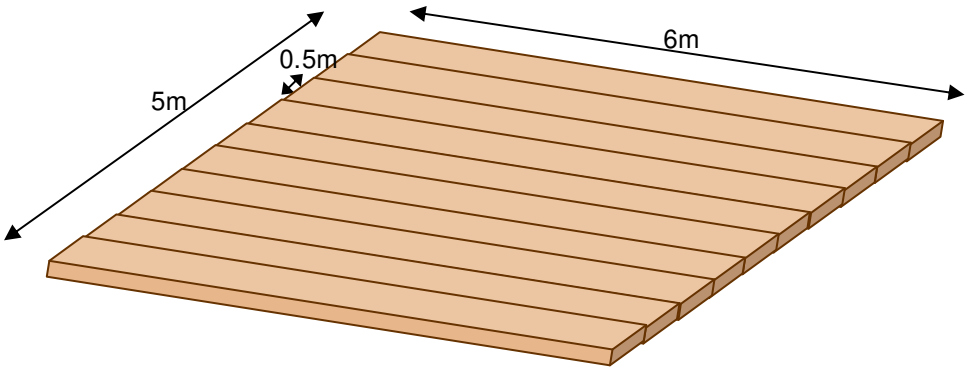
Advanced worksheet

In buildings, beams are put together to create floors. A team of engineers is constructing a floor for a 5m by 6m room. The floor is made up of 10 beams which are each 0.5m wide and 6m long.



Using the information below, and the algebraic expressions for beam deflection on the previous sheets, work out how thick the beams need to be and find the cheapest price for the floor.

The architect has chosen a wood for which k= 0.0002.

To be safe, the beams must deflect no more than 0.003m.

It is important to know the greatest weight each beam must be able to bear.Imagine there is one 125kg rugby player on each square metre of the floor. The rugby players don't stay still; they walk around in the room. Sometimes there will be no players on one beam but lots on another. What do you think is the maximum number of players that can fit on one beam at one time?

Building regulations demand a **safety margin** of 1.5. That means that the beams must be 50% thicker than they need to be.

There are 3 suppliers that sell suitable wooden beams:

At **Anderson's** it costs £200 for a 0.2m thick beam and £30 for each additional centimetre of depth.

At **Boonville's** it costs £550 for a 0.3m thick beam and £20 for each additional centimetre of depth.

At **Clark and Carr** it costs £20 for each centimetre of depth.