Coordinates

more maths grads
multiplying opportunities

Starter worksheet

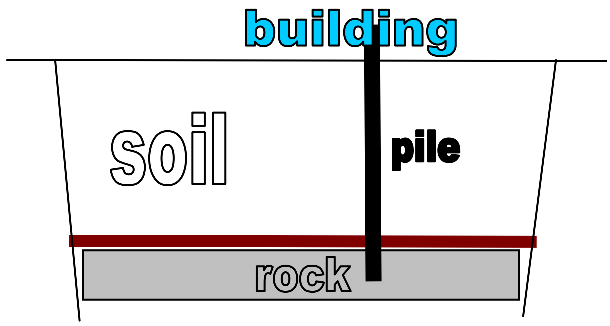
How do you make sure buildings stay up?

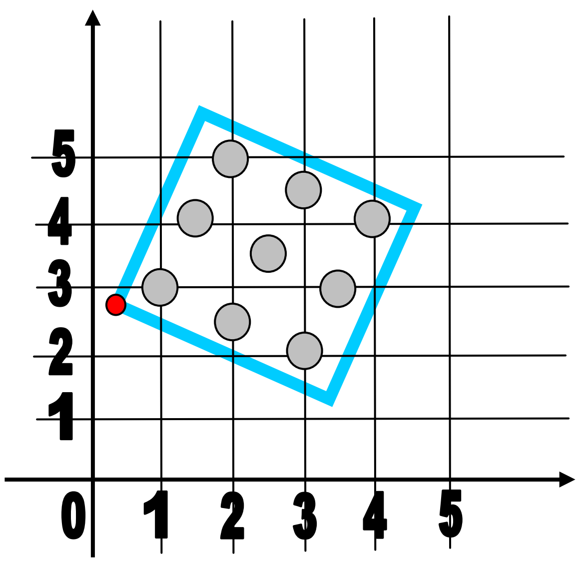
If you're building a twenty floor skyscraper, you want to be completely sure it won't fall down. It's important to make sure that it has a proper foundation. Suppose that your building is in an area where there's a layer of rock about five metres down, but above that there's only poor quality soil that won't support the weight of the building. Then you need to put some **piles** in to support the building.

Piles are long and thin. They might have a cuboid or cylinder shape. They can be made of steel or timber or concrete and are pushed through the soil into the rock. The important thing is that they allow the building to be supported by them, and they stand on the firm rock below the soil.

When pushing (engineers call it **driving**) piles into the ground, you have to make sure that they're spaced out evenly.

Imagine that you're in a helicopter looking down on the building site. You could make a map of the site and put coordinates on it so that you can say where the piles should go.





In the grid shown, there are nine positions where the piles are to go, marked with grey dots. Two of these positions are (1,3) and (2, 2.5). Where are the other seven positions?

The building is marked as a blue rectangle on the grid. If we made the building twice as long in each direction, keeping one corner fixed at the red dot, how many more piles would we need? Where should they be put?

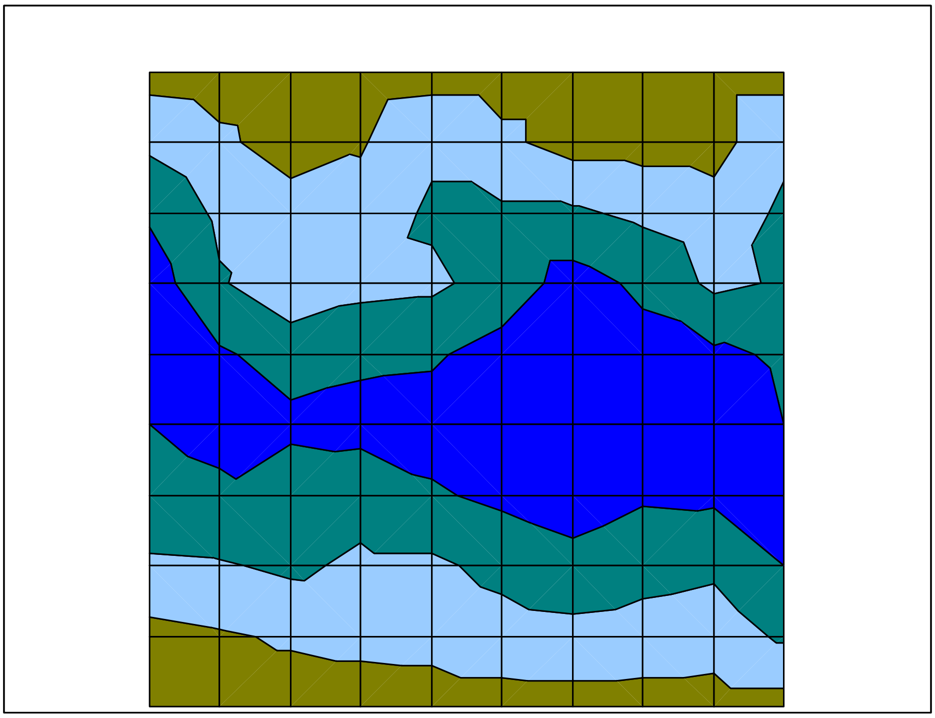
Coordinates

more maths grads
multiplying opportunities

Core worksheet

Building bridges

An engineering company has been asked to build a bridge across the river shown. The grid can be given coordinates so that the bottom left hand corner is (0,0) and the top right hand corner is (9,9). The foundation piles for the bridge are to be placed at the following coordinates:



(1,0), (2,0), (3,0)
(1,1), (2,1), (3,1)
(1,2), (2,2), (3,2)
(1,3), (2,3), (3,3)
(1,4), (2,4), (3,4)
(1,5), (2,5), (3,5)
(1,6), (2,6), (3,6)
(1,7), (2,7), (3,7)
(1,8), (2,8), (3,8)
(1,9), (2,9), (3,9)

1. Draw where the piles for the bridge should go.

2. The colour shows where land is, or the depth of the water. For example, there is:
   Land at (2,8)
   Shallow water at (2,7)
   Deep water at (2,5)
   Very deep water at (2,4)
   Suppose that it costs £2000 to drive a pile into land, £3500 to drive a pile into shallow water, £4500 to drive a pile into deep water, and £5500 to drive a pile into very deep water.
   How much will it cost to drive all of the piles for the bridge, in total?

3. An earlier suggestion had been to run the bridge from (5,0), (6,0) and (7,0) directly across to (5,9), (6,9) and (7,9). How much more expensive would it have been to drive the piles for this route?

Scale Factors

Starter worksheet

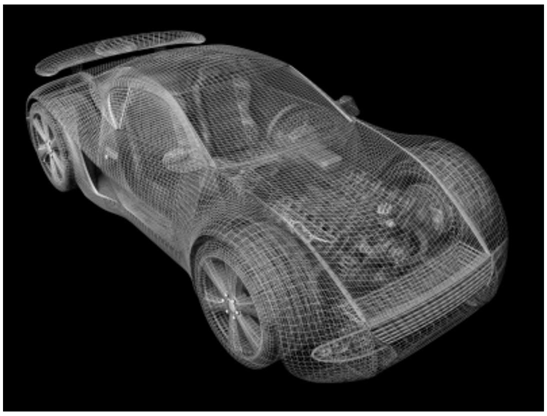
Where do people use scale factors?

People tend to use scale factors when they want to make a **scale model** of something. Usually this is when they're trying to understand something which is either very big or very small.

When an architect or engineer is working on the design and construction of a large building they often make a scale model of the building. They might use computer software to make a virtual model or they might make a physical model that can be placed on a desk. In either case the dimensions of the building – its height, width and length – have to be **scaled** to the proportions of the model.

Similarly, a medical engineer might make a model of a part of the body like the heart which is bigger than an actual heart, so that trainee doctors can see how each of the bits of it work.

Car designers also use computer scale models of cars so that they can see how all the different bits fit together. To make a model car, someone has to work out how to scale down all of the sizes so that the small model car looks just like the real car – only smaller!



Suppose that you work for a firm that makes model cars. You've been told that each model car has to look exactly like the real car but with a scale factor of 1 : 24.

This means that if the real car is 2.4 metres long, the length of the model car can be calculated by dividing 2.4 metres by the scale factor 24. This is 0.1 metres.

Therefore the model car is 10 centimetres long.

1. Suppose that the wheels of the real car are 45 centimetres in diameter. What is the diameter of a wheel of the model car? ______________

2. Suppose that the steering wheel of the real car is 30 centimetres in diameter. What is the diameter of the steering wheel of the model car? ______________

3. The driver's seat in the real car needs 2 square metres of fabric to cover it. How many square centimetres of fabric is this? ______________

4. Your colleague Philip tells you that the model car needs about 35 square centimetres of fabric to cover the driver's seat. Why does he say this? Do you agree with him?

   Explain your answer here.

Scale Factors

more maths grads
multiplying opportunities

Core worksheet

Creating a scale map of the United Kingdom for use in a weather forecast

If you buy a map it's important to get the right scale so that you can see the level of detail you need.

A street map of your local area might have a scale factor of 1:20000. That means that 1 centimetre on the street map represents 20000 centimetres, which is the same as 200 metres.

A national road map might have a scale factor of 1:250000. In such a road map, 1 centimetre represents 2.5 kilometres.

Weather forecasters on television or weather sites on the internet use a scale map of the UK to show us what the weather will be like in different parts of the country.

Suppose that you're looking at a weather map of the UK on a computer screen, where the UK looks just like the map in the picture shown.

The UK mainland is just under 500 kilometres from east to west and just under 1000 kilometres from north to south. On the screen the mainland appears to be 16 centimetres from north to south. The whole map is 20 centimetres in height by 14 centimetres in width.

1. Approximately, what is the scale of the map on the computer screen?

   ____

2. The length of the UK coastline is about 12500 kilometres.
   If you measured the length of the coastline on the screen, approximately how long would it be?

   ____

3. Whenever you scale a drawing on a computer it has to use this mathematics to make sure that the pixels on the screen display the correct colours. Many computer screens have a resolution of 96 dpi (dots per inch), which is about 37.8 pixels per centimetre, or 37.8 x 37.8 = 1429 pixels per square centimetre.

   Approximately how many pixels are needed to display the 20cm x 14cm map on the screen?

   ____

   Estimate the area that each pixel represents, giving your answer in square kilometres to the nearest whole number.

   ____

Angles

Starter worksheet

Penalty shoot out

World Cup football games end with a penalty shootout if the teams are still level on goals at the end. Each team has to send out players in turn to kick a ball from a fixed spot into the goal.

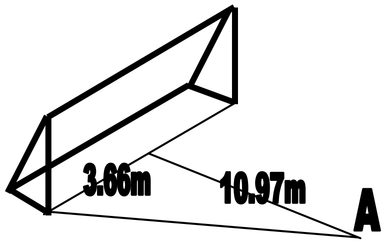
If you were the player taking a critical penalty kick that would either give your team victory in the World Cup final or send them home in despair, you'd want to have the best chance of getting your kick past the opposing goalkeeper and into the goal!

It's very hard for a goalkeeper to reach the top left and right corners of the goal. But it's also very difficult to kick the ball accurately into those corners. One of the reasons for this is that a small difference in the **angle** that you kick the ball at makes a huge difference in the height of the ball above the ground when it reaches the goal.

A few degrees wrong in the angle one way and you'll send the ball way above the bar and into the crowd. A few degrees wrong in the angle the other way and the ball will pass low enough for the goalkeeper to be able to catch it easily if they dive that way.

In international football, the goalposts are 24 feet apart and the crossbar connecting them is 8 feet above the ground. You take a penalty kick from a spot 36 feet away from the spot halfway between the goalposts.

Let's convert these figures to metres using the conversion factor 1 foot = 0.3048 metres. In metric units, the goalposts are 7.32 metres apart, the crossbar is 2.44 metres above the ground and the penalty spot is 10.97 metres back.



If you're aiming for the corner of the goal you've got to kick a horizontal distance which is equal to the third side of the triangle shown. It can be calculated using Pythagoras' theorem: it's approximately 11.56 metres.

1. Look at the angle at corner A of this triangle. Using any method you prefer, estimate this angle to the nearest degree. The diagram is not to scale!

   ____

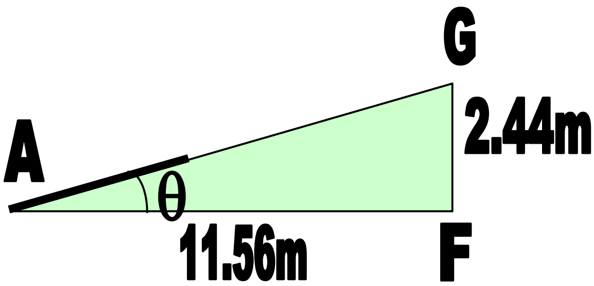
2. Suppose that you take the risky shot and aim your kick along the bold line.
   Assuming you can aim your kick accurately, the ball will be heading for the left goalpost.
   Let's assume also that you can kick the ball so that it covers 20 metres per second.

(a) If you kick the ball straight up into the air, how long will it take to reach the goal?

(b) If you kick the ball horizontally along the ground, how long will it take to reach the goal?

Angles

Core worksheet



What's the right angle to kick the ball at?

The triangle shows the angle involved in taking a penalty kick into the top left hand corner of the goal.

The right hand side represents the height of the goal, which is 8 feet, or 2.44 metres.

The horizontal side represents the distance to the corner of the goal, which is 11.56 metres.

1. If you stand at A, what is the angle **θ** between the foot of the goalpost at F and the top of the goalpost at G? (There are several ways to answer this. One would be to draw an accurate scale diagram of the triangle and then measure the angle.)

   ____

2. Suppose that you kick the ball directly at G, so that it starts off heading straight towards G. While it is in flight it will be affected by gravity pulling it down towards the ground. Will the ball hit the line FG below or above the point G?

   ____

3. Suppose that you want to kick the ball so that it hits G, the top of the goal post. Given your answer to Question 2, should you kick the ball at angle **θ** or at a smaller or larger angle? Explain your answer.

   ____

   ____

Angles

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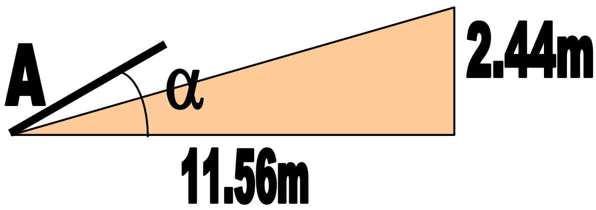
Advanced worksheet

Project

You meet up with Nadia, who's studied mechanics at A-level. She tells you that if you kick the ball at angle α and at 20 metres per second then the height **y** of the ball when it reaches the goal can be modelled by the following equation.



$$y = 11.56 \tan\alpha - \frac{1.637}{(\cos\alpha)^2}$$

If this gives you a negative answer it means that the ball bounced on the ground at least once before it reached the goal. If it gives a positive answer then that's the height above the ground.

Suppose that you kick the ball at the following angles α and use the equation to find out the height of the ball when it reaches the goal. Remember if it's over 2.44m then it's too high.

Angle α (in degrees)	Height **y** (in metres)	Did the ball bounce?	Was it too high?
5 degrees			
10 degrees			
15 degrees			
20 degrees			
25 degrees			
30 degrees			

Choose angles for the final four rows of the table to see if you can find the perfect angle to hit the top of the goalpost to the nearest degree. How does this compare with the angle **θ**?

What other factors would affect the angle you need to kick the ball at?

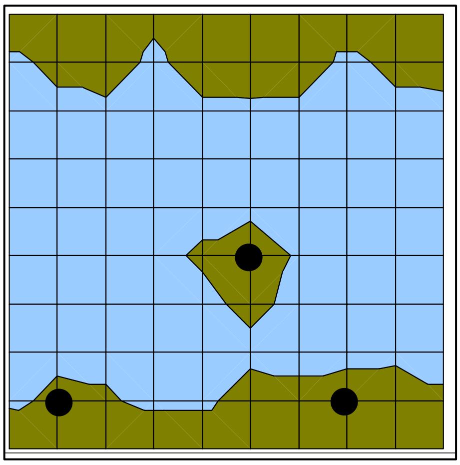
Surveying with trigonometry

more maths grads
multiplying opportunities

Starter worksheet

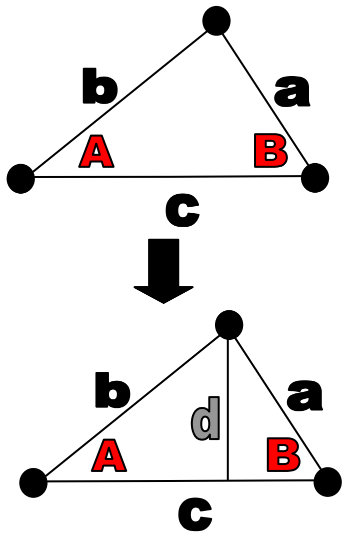
If you're an engineer trying to design a bridge to cross a river, you have to be able to say exactly where the construction company should put each of the parts for the bridge. One way of doing this is to use GPS receivers which communicate with satellites above the earth to work out exactly where on the Earth's surface the receiver is. GPS stands for Global Positioning System and is a widely used technology. But even if you have a GPS receiver, you may need to find out the position of somewhere that you can't easily get to.

Suppose that you're standing by a wide river which has an island in the middle of it.



You need to find out the distance from each of the two points marked on the river bank to the point marked on the island. But you can't get to the island easily.

Suppose that you can measure the distance **c** between the two points on the river bank. You can also measure two of the angles in the triangle that these three points make: **A** and **B**.



1. One way to work out the distances **a** and **b** is to split the triangle up into two smaller right angled triangles. Then we have

$$c = \frac{d}{\tan A} + \frac{d}{\tan B}$$

and we can rearrange this to work out an expression for **d** if we know **A**, **B** and **c**. Write down this expression.

2. Write down an expression for **b** in terms of **A** and **d**.

3. Write down an expression for **a** in terms of **B** and **d**.

4. Suppose that **A** is 30 degrees, **B** is 60 degrees and **c** is 5 kilometres. Calculate **d** and therefore **a** and **b**.

5. Suppose that **A** is 25 degrees, **B** is 75 degrees and **c** is 3 kilometres. Calculate **d** and therefore **a** and **b**.

6. Suppose that **A** is 45 degrees, **B** is 65 degrees and **c** is 7 kilometres. Calculate **d** and therefore **a** and **b**.

7. Write down some problems that might occur if you were a surveyor trying to carry out this task in a practical situation.

Surveying with trigonometry

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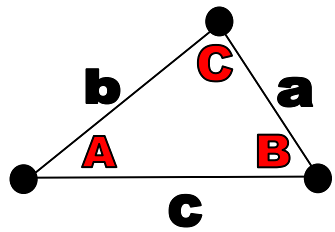
multiplying opportunities

Core worksheet

Surveying distances

If you need to work out the distances **a** and **b** when you know the angles **A** and **B** and the distance **c**, you can split the triangle into two smaller right angled triangles provided that **A** and **B** are both acute angles (both less than 90 degrees).



A different way of calculating **a** and **b** is to use the sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

We also know that the third angle **C** of the triangle satisfies **C** = 180 – (**A**+ **B**) since angles of a triangle add up to 180 degrees.

1. To calculate **a** we need to take the first and third parts of the above equation and rearrange them to give an expression for **a** in terms of **A**, **B** and **c**. Write down this expression.

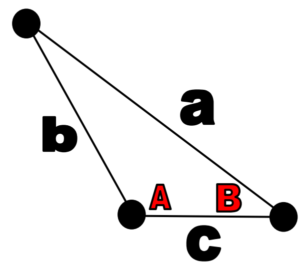
2. Write down the corresponding expression for **b** in terms of **A**, **B** and **c**.

3. Suppose that **A** is 30 degrees, **B** is 60 degrees and **c** is 5 kilometres. Calculate **a** and **b** directly.

4. Suppose that **A** is 25 degrees, **B** is 75 degrees and **c** is 3 kilometres. Calculate **a** and **b** directly.

5. Suppose that **A** is 45 degrees, **B** is 65 degrees and **c** is 7 kilometres. Calculate **a** and **b** directly.

6. The advantage of this method is that it works even when the triangle can't be split up into two smaller right angled triangles. Suppose that **A** is 105 degrees, **B** is 35 degrees and **c** is 2 kilometres. Calculate **a** and **b** directly.



7. Suppose that **A** is 125 degrees, **B** is 15 degrees and **c** is 3 kilometres. Calculate **a** and **b** directly.

8. Make a sketch of the sine curve on a graph by putting the angle **A** along the horizontal axis and sin **A** along the vertical axis.

9. For **A**, **B** between 0 and 180 degrees, which of the following rules is always correct?
   A. sin (180 – (**A**+ **B**)) = - sin (**A**+ **B**)
   B. sin (180 – (**A**+ **B**)) = sin (**A**+ **B**)
   C. sin (180 – (**A**+ **B**)) = 1 - sin (**A**+ **B**)
   D. sin (180 – (**A**+ **B**)) = 1 / sin (**A**+ **B**)

10. Use the correct rule to write down simplified expressions for **a** and **b**.

Sine waves in music

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Starter worksheet

Keyboard musical note	Frequency
F2	87.31
F2#	92.50
G2	98.00
G2#	103.83
A2	110.00
A2#	116.54
B2	123.47
C3	130.81
C3#	138.59
D3	146.83
D3#	155.56
E3	164.81
F3	174.61
F3#	185.00
G3	196.00
G3#	207.65
A3	220.00
A3#	233.08
B3	246.94
C4	261.63
C4#	277.18
D4	293.66
D4#	311.13
E4	329.63
F4	349.23
F4#	369.99
G4	392.00
G4#	415.30
A4	440.00
A4#	466.16
B4	493.88
C5	523.25
C5#	554.37
D5	587.33
D5#	622.25
E5	659.26
F5	698.46
F5#	739.99
G5	783.99
G5#	830.61
A5	880.00
A5#	932.33
B5	987.77
C6	1046.50

The decimal numbers are the frequencies of those notes in Hertz (the number of peaks of the sound wave in a single second).

The table gives the names and frequencies of various musical notes that you would find on a piano or keyboard. C4 is middle C and is a note that almost all female singers and most male singers can sing comfortably.

The white keys are called A, B, C, D, E, F, G then this pattern repeats again. The lowest note called A in this table is called A2, then the next time we come to a note A it's called A3, and so on. A grand piano keyboard would have keys from A0 to C8 on it.

The black keys are called A#, C#, D#, F# and G#, where the # symbol is called "sharp" and means that that note is slightly higher. So C4# is a slightly higher note than C4.

Look at the keyboard and answer the following questions.

1. If you divide the frequency of A5 by the frequency of A4, what do you get?

2. Which other pairs of frequencies give this result when you divide one by the other?

3. If you divide the frequency of A4# by the frequency of A4, what do you get?

4. Which other pairs of frequencies give this result?

5. What would the frequency of C7 be?

6. What would the frequency of C6# be?

7. Which note has frequency 3520 Hertz?

Sine waves in music

more maths grads
multiplying opportunities

Core worksheet

Keyboard note	Frequency	Frequency divided by 261.63
C4	261.63	
C4#	277.18	
D4	293.66	
D4#	311.13	
E4	329.63	
F4	349.23	
F4#	369.99	
G4	392.00	
G4#	415.30	
A4	440.00	
A4#	466.16	
B4	493.88	
C5	523.25	

Any pair of notes where one note's frequency is exactly double the other note's frequency are said to be an **octave** apart. The two notes sound good when played together.

The Greek mathematician Pythagoras is famous for his theorem about triangles, but he is also thought to have discovered that notes which sound good together tend to have frequencies whose ratio is a fraction involving two small whole numbers.

Complete the third column of the table and use it to answer the following questions.

1. Try to find two notes so that the frequency of one of them is double the other.

2. Which note has frequency which is close to being 3/2 of the frequency of C4?

3. Which note has frequency which is close to being 4/3 of the frequency of C4?

4. Which note has frequency which is close to being 5/3 of the frequency of C4?

5. Is C4# / C4 close to a fraction involving two small whole numbers?

Sine waves in music

more maths grads
multiplying opportunities

Advanced worksheet

Notes on a piano or keyboard are an approximation to the frequencies that would really sound best together. For example, think about G4 (the G above middle C) and suppose that you were trying to work out the best frequency to tune that note to.

- G4 sounds best when played with C4 if its frequency is exactly 3/2 times 261.63, which would be 392.45.
- But it sounds best when played with D4 if its frequency is exactly 4/3 times 277.18, which would be 391.55.

You could change the frequency of D4, but then some other pair of notes would sound worse. On a violin or many other instruments, musicians play slightly different frequencies for G4 depending on the other notes that are being played at the time.

1. Calculate the ratio of frequencies between any pair of consecutive notes on the keyboard.

2. Do all pairs of consecutive notes have this ratio?

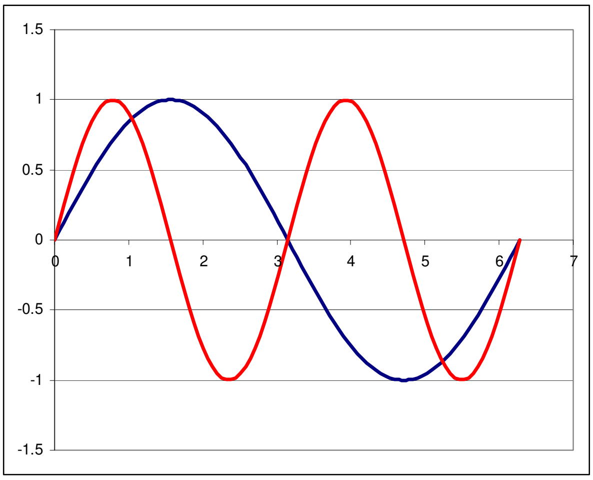
3. Take your answer to question 1, calculating it to three decimal places, and multiply twelve copies of this number together. Round your answer to one decimal place.

4. Explain how keyboard and piano makers solve the problem described above, using your calculations to help you.

Why do notes sound better if their frequencies are in a ratio like 3:2 or 4:3?

Pure notes are made by compressing air in a way that can be modelled by a sine wave – peaks of the wave mean the air is compressed a lot, troughs mean the air molecules are spread out. The waves **sin x** and **sin 2x** shown in the picture match up in several places. Notes an octave apart like C4 and C5 can be represented by sine waves like these: C4 is the blue curve and C5 the red curve. Notes where the sine waves don't match up well sound worse when played together.

In fact a real musical instrument uses a lot of other tricks. On a piano playing the key middle C will not only let you hear the pure tone with 261.63Hz but also – more quietly! – other tones such as C5 and G5. These are known as **harmonics**, and musical instruments sound different from each other partly because they all use harmonics in slightly different ways.



Algebraic Expressions

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multiplying opportunities

Starter worksheet

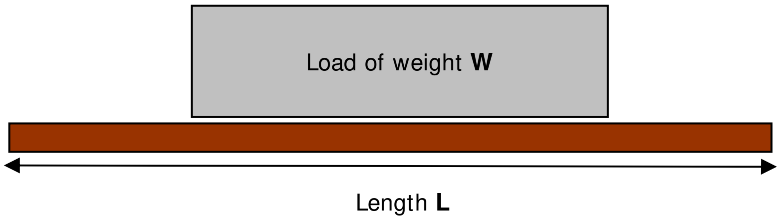
Suppose we have a wooden beam with a heavy weight on it.

As we put heavier and heavier weights on the beam, it will bend more. Eventually it may break. Suppose that you're designing a building with several storeys. You would need to decide how thick the floor needs to be. You can't just construct the building and then test it by holding a big party on the top floor – it's too dangerous.

"When we're designing a building, we imagine that there's a heavy person – like a rugby player or a super heavyweight boxer – standing on each square metre of the floor. We work out how thick the floor needs to be to hold all these people without bending by more than a few millimetres. Then we make the floor even thicker than that. There are standard legal safety margins which we work with."

- *Chris Bean, Structural Engineer*

How do the engineers work out how thick the floor has to be? They use some simple algebra.



For a beam of length **L** metres with a load of weight **W** kilograms sitting on it, the deflection **D** is the amount in metres that the beam bends.

Scientific testing shows that there's an equation which relates **D**, **W** and **L**:

$$D = c \times W \times L^3$$

where **c** is a constant depending on things like the type of wood used, how thick the floor is and whether the weight on the beam is all at one point or spread evenly along the beam.

Calculate the deflection **D** in each of the following cases.

1. **c** = 0.0001, **W** = 10 kg, **L** = 2 m
2. **c** = 0.0001, **W** = 50 kg, **L** = 2 m
3. **c** = 0.0001, **W** = 50 kg, **L** = 3 m
4. **c** = 0.0001, **W** = 150 kg, **L** = 3 m

What do you observe? Write down three things that you notice.

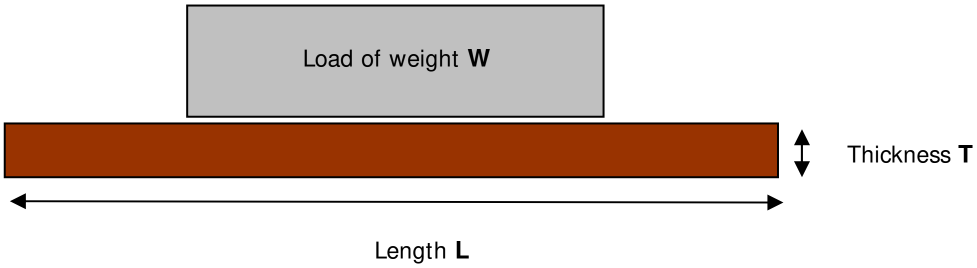
(For example, does the beam bend more with the same weight if it is longer?)

Algebraic Expressions

more maths grads
multiplying opportunities

Core worksheet

For safety, beams in a building have to cope with heavy weights – just in case an international rugby team comes to visit! They also have to be the right length for the building design. This means that if a team of engineers calculate that heavy weights (or heavyweights!) will cause a beam to deflect too much, they can't just make the beam shorter. Instead, they can try to decrease the deflection by making the beam thicker.

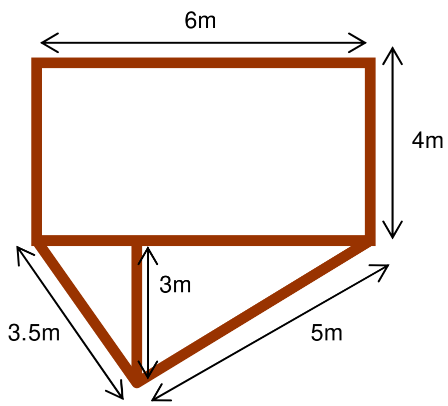


For a beam of length **L** metres and thickness **T** metres with a load of weight **W** kilograms sitting on it, the deflection **D** is the amount in metres that the beam bends. The equation we have been using to calculate the deflection is $\mathbf{D = c \times W \times L^3}$.

This is not very useful if we want to see how the thickness of the beam affects the deflection. However, the constant **c** can be calculated using another equation involving a number **k** whose value depends on the type of wood.

$$c = \frac{k}{900000\,T^3}$$

Use this to write a new equation for D.



The diagram shows the structure an engineer wants to build. A beam is **safe** if it deflects no more than 0.002m when holding 1500kg.

The rectangle is built out of one type of wood, where k=0.0001. Calculate how thick each must be to be safe:

1. the 6m beam
2. the 4m beam

The other beams are made of a different type of wood, where k= 0.0002. For each beam, work out if it is safe. If it is not safe, how thick does it need to be?

3. The 3m beam is 0.13m thick.
4. The 3.5m beam is 0.23m thick.
5. The 5m beam is 0.25m thick.

Algebraic Expressions

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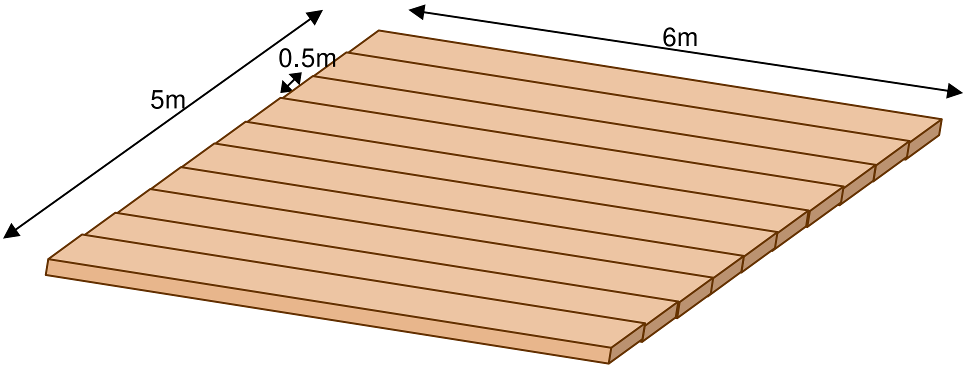
Advanced worksheet

In buildings, beams are put together to create floors. A team of engineers is constructing a floor for a 5m by 6m room. The floor is made up of 10 beams which are each 0.5m wide and 6m long.



Using the information below, and the algebraic expressions for beam deflection on the previous sheets, work out how thick the beams need to be and find the cheapest price for the floor.

The architect has chosen a wood for which k= 0.0002.

To be safe, the beams must deflect no more than 0.003m.

It is important to know the greatest weight each beam must be able to bear.Imagine there is one 125kg rugby player on each square metre of the floor. The rugby players don't stay still; they walk around in the room. Sometimes there will be no players on one beam but lots on another. What do you think is the maximum number of players that can fit on one beam at one time?

Building regulations demand a **safety margin** of 1.5. That means that the beams must be 50% thicker than they need to be.

There are 3 suppliers that sell suitable wooden beams:

At **Anderson's** it costs £200 for a 0.2m thick beam and £30 for each additional centimetre of depth.

At **Boonville's** it costs £550 for a 0.3m thick beam and £20 for each additional centimetre of depth.

At **Clark and Carr** it costs £20 for each centimetre of depth.

Simultaneous Equations

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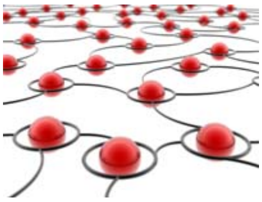
Starter worksheet

Simultaneous equations are a branch of algebra where you may be asked to solve problems such as the following.

Anna goes to the shop and buys three apples and two pears, which costs her £1.46. Bilal goes to the same shop and buys one apple and four pears, which costs him £1.52. How much does an apple cost?



The first question you may find yourself asking is – why don't you go to the shop yourself and find out? It's not usual in real life to have to solve this kind of problem. But this kind of problem – on a much more complicated scale! – is solved by companies running web search engines all the time. When you type in "the world's tallest man" to your favourite search engine, how does it work out which sites to show you? We'll look at how that works on the next sheet.

The second question you should ask is how you can solve this problem of apples and pears, so that you can understand the maths underlying internet searches.

There are lots of ways to solve the problem. Here are a few ways.

Method 1. (Reasoning in words)
Suppose that Bilal had bought three times the quantity of fruit that he bought – three apples and twelve pears. Then he'd spend three times £1.52, which is £4.56. Now Anna's three apples and two pears cost £1.46, so the difference between these will be the cost of ten pears. Since £4.56 - £1.46 = £3.10, a single pear costs 31p. Therefore four pears cost £1.24, so the apple Bilal bought must cost £1.52 - £1.24, which is 28p.

Method 2. (The same reasoning, but with algebraic symbols)
All of the above working can be put into an algebraic form.
Let a be the cost of an apple and b the cost of a pear, in pence.
Therefore we have two equations: $3a + 2b = 146$ and $a + 4b = 152$.
Multiply the second equation by 3 on both sides to give $3a + 12b = 456$.
Subtract the first equation from the second as follows: $3a + 12b – (3a + 2b) = 456 – 146$.
Simplify to give $10b = 310$ and divide by 10 on both sides to give $b = 31$.
Then substitute $b = 31$ into $a + 4b = 152$ to give the equation $a = 152 – 4x31 = 28$.
So an apple costs 28p.

Method 3. (Estimation)
Notice that both Anna and Bilal bought five pieces of fruit, and that the total costs were very similar: £1.46 and £1.52. Therefore each piece of fruit must cost roughly £1.50 divided by 5, or 30p. Try different values for the cost of an apple and the cost of a pear until you see that 28p and 31p works.

1. Find another way of working out the cost of an apple. Which method do you prefer?

2. Nadia goes to the shop and buys three bananas and a mango, which costs her £1.89. Oliver goes to the same shop and buys eight bananas and two mangos, which costs him £4.28.
(a) How much does a mango cost? Use any method you like to calculate the answer.
(b) Use a different method to calculate the cost of a mango.

Simultaneous Equations

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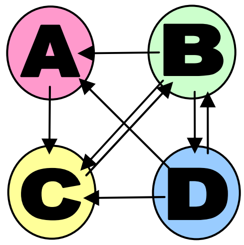
Core worksheet

Simultaneous equations aren't just used to figure out the cost of apples and bananas. If you use an internet search engine to find websites then you're using the results of solving simultaneous equations. However these calculations involve millions of equations rather than just two or three. Although there are powerful computers to do the hard work, they need to be programmed to carry out the right algorithm. Understanding **how** to solve the equations in the fastest way possible makes all the difference for companies running a search engine: being very successful or going out of business.

To understand a bit about how search engines work, we'll look at one simplified example. Suppose that the whole internet consists of just four pages. In the diagram, an arrow means that the webpage at the start of the arrow contains a link to the webpage at the end of the arrow. So the webpage British Beaches has a link to Amazing Adventures but not the other way around.



Webpage A: Amazing Adventures **Webpage B:** British Beaches
Webpage C: Caribbean Cruises **Webpage D:** Dixon's Directory

A good webpage will usually be connected to by lots of links from other webpages. This isn't the only thing that search engines look at when deciding which webpages to suggest, but it's a start.

1. How many links point to each of the above webpages? Which webpage has the most links to it?

Search engines take this idea one step further and give webpages a higher value if they are linked to by webpages which themselves have a higher value. The next two questions give an idea of how this is done.

2. In the grid on the left below, write 1 in row A and column B if there's an arrow pointing from B to A, and write 0 if there isn't an arrow from B to A. Do the same for all the boxes.

The grid on the right is calculated by taking each column of the first grid in turn and dividing each of the entries by the total gained by adding up the numbers in that column. Check this against your answer to Q2.

	A	B	C	D
A	0			
B		0		
C			0	
D				0

	A	B	C	D
A	0	$^1/_3$	0	$^1/_3$
B	0	0	1	$^1/_3$
C	1	$^1/_3$	0	$^1/_3$
D	0	$^1/_3$	0	0

3. This grid will help us work out a value for each webpage. Take four variables **a**, **b**, **c** and **d**. These will have values between 0 and 1, and are the quality values calculated for Webpage A, B, C and D respectively. Take four simultaneous equations so that the coefficients come from the grid.

   First row: $1/3 \times b + 1/3 \times d = a$ Second row: $c + 1/3 \times d = b$
   Third row: $a + 1/3 \times b + 1/3 \times d = c$ Fourth row: $1/3 \times b = d$

   Assuming that $a + b + c + d = 1$, solve the equations to find the values of **a**, **b**, **c** and **d**. If the search engine lists the pages in order of their value with largest first, what order will they be listed in?

Quadratic Equations in Sport

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Starter worksheet

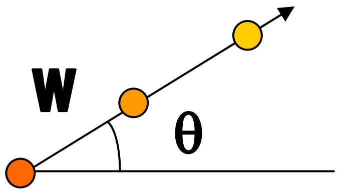
Lots of sports involve kicking, hitting or throwing a ball so it hits a target: basketball, football, golf, netball, rugby, shotputting, squash, table tennis, tennis, volleyball… and more!

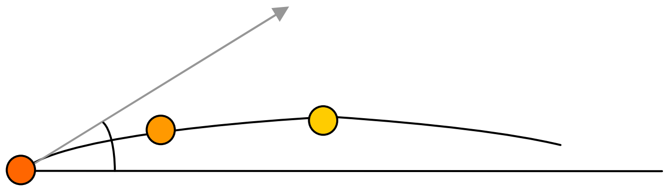
What all of these sports have in common is that practice makes perfect: to get the ball to hit the target you have to throw or kick it at the right speed and angle. But thinking about the maths involved can be useful! It can help you understand whether you need to be more precise with the angle or whether you need to work out in the gym so you can hit the ball at a higher speed.

Suppose you throw a ball with speed **w** and at an angle **θ**. Just think about how the height of the ball varies. If the angle is close to zero, the ball will go up a little bit and then start to fall to the ground. If the angle is close to 90 degrees, the ball will go up a lot but it will land close to where you started.

It turns out that if you want to get the ball as far as possible before it lands, you need to throw it with an angle close to 45 degrees, and as hard as you can.

The ball follows a path which can be written down as a quadratic equation. At the start the ball travels at angle **θ**, but then gravity begins to pull it down and it travels at shallower (and smaller) angles. We're assuming that there isn't anything else affecting the ball, so for example it isn't being blown around by a strong wind.





Let's think about quadratic equations and what they look like.

Quadratic equations have the form $\mathbf{y = ax^2 + bx + c}$ where **x** and **y** are variables and **a**, **b** and **c** are coefficients. For any particular equation this means that we can imagine that **a**, **b** and **c** are fixed numbers. The graph of the curve shows how the value of **y** changes if we change **x**.

If **a** is a negative number then the graph of the curve will go up to a maximum point and then down.
If **a** is a positive number then the graph of the curve will go down to a minimum point and then up.

1. What happens if **a = 0**?

2. Think about the curve $\mathbf{y = 10 - x^2}$ for **x** between the values **x = -5** and **x = 5**.
   (a) In this equation **b = 0**. What are the values of **a** and **c**?
   (b) What is the maximum possible value of **y** satisfying the equation?
   (c) What is the minimum possible value of **y** for **x** in the range given?
   (d) Sketch the curve for **x** in the range given.

3. Suppose that we want an equation that passes through the point where **x=0** and **y=0**.
   What value of **c** in the equation $\mathbf{y = ax^2 + bx + c}$ do we need for this to happen?

Quadratic Equations in Sport

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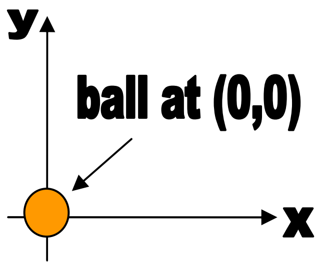
Core worksheet

If you hit a ball into the air – like kicking a football towards the goal or a tennis ball back towards your opponent - and want to calculate exactly where it will land, you need a quadratic equation. In real life you wouldn't be out there with a calculator, but if you were programming a sports game you'd have to give the computer some equations so it could figure out how to make the gameplay feel real.

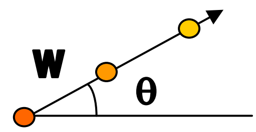
We can use a two-dimensional coordinate system with **x** measuring the horizontal distance along the ground and **y** the vertical distance up into the air.



Suppose that the point (0,0) – where **x=0** and **y=0** - is the point where we hit or kick the ball. The equation $\mathbf{y = ax^2 + bx}$ will always pass through the point **x=0**, **y=0**.

If we let **x** be the horizontal distance the ball travels, and **y** be the vertical distance, then **x** and **y** are related by a quadratic equation. The equation depends on a number **g** which measures the pull of gravity, on **w** which is the speed you hit or kick the ball at, and on **θ** which is the angle you hit or kick it at.

$$y = \frac{-g}{2w^2(\cos\theta)^2}x^2 + (\tan\theta)\,x$$



The equation is given above. It may look a bit complicated but it's really just $\mathbf{y = ax^2 + bx}$ with **a** and **b** chosen to match the situation. Let's work through some examples. Suppose that **x** and **y** are measured in metres, the speed **w** in metres/second, and that gravity **g** is 9.8 m/s^2. Assume we kick or hit the ball from ground level.

1. The vertical distance is zero at the start, when **x** = 0 and **y** = 0. It's also zero when the ball hits the ground again. In terms of **w**, **θ** and **g**, what is **x** when **y** = 0 for the second time?

   (Hint: This is just like solving 0 = $\mathbf{ax^2 + bx}$ for **x** in terms of **a** and **b**.)

2. Suppose that **g** = 9.8 m/s^2 and that **w** = 10 m/s^2. By subsituting these values into the equation above, write down the equation for the path of the ball when
   - (a) **θ** = 5 degrees
   - (b) **θ** = 30 degrees
   - (c) **θ** = 45 degrees
   - (d) **θ** = 60 degrees
   - (e) **θ** = 85 degrees

3. For each of the angles in Question 2, how far does the ball travel before it reaches the ground?

4. Of the five angles tested, which allows the ball to be thrown or kicked the furthest distance?

5. How else could you get the ball to go further?

Business equations

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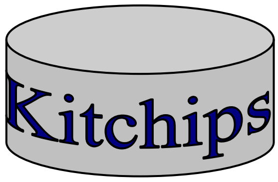
Starter worksheet

You work for a company called Kitchips that makes nutritious cat food. They want new packaging for their products and have created three new packaging ideas. Each idea affects the total cost of making a tin of cat food. You will have to decide which idea to use and what price to sell the tins at.



Basic Design tins (in black and white) overall cost 42p each to produce. Standard Design tins (in colour) cost 50p each overall. Distinctive Design tins (in deluxe colour) cost 68p each overall.

Kitchips has also commissioned a Sales Survey to find out how many people would buy each tin design at what price.

> Results of the Sales Survey:
>
> If the price is p pence, the number of people who would buy each tin could be modelled by 50000-300p for Basic Design, 50000-200p for Standard Design and 50000-150p for Distinctive Design.

Decide what packaging design you want, the price to sell one tin at and calculate your profit. Can you work out the largest possible profit you could make?

Packaging	Production cost of one tin	Price to sell the tin at	Number of people buying	Total profit made

Business equations

Core worksheet

You work for a company called Kitchips that makes nutritious cat food. They are bringing out a new product and you will have to decide how the product should be sold and at what price.

It costs 50p to produce a tin of Basic Mix cat food, 58p to produce a tin of Superior Mix, and 66p to produce a tin of Top Quality Mix.

Results of the Sales Survey

Kitchips commissioned another survey to find out how many people would buy each type of mix at what price. The conclusions were:

If the price was p pounds, the number of people who would buy each type of mix could be modelled by $30000 \div p^3$ for the Basic Mix, $40000 \div p^3$ for the Superior Mix and $50000 \div p^3$ for the Top Quality Mix.

Decide what type of mix you want to sell, the price to sell it at and calculate your profit. Can you work out the largest possible profit you could make?

Type of Mix	Production cost of one tin	Price to sell the tin at	Number of people buying	Total profit made

Calculating the Weather

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Starter worksheet

Meteorologists use many different techniques and formulae to calculate and predict the weather. But one part of maths that they have to use a great deal is **percentages**.

When water evaporates it turns into **vapour** in the air. The amount of water vapour in the air is changes at different temperatures and in different places.

Can you think of somewhere that might have a tiny amount of water in the air? What about a place with lots of water in the air?

Meteorologists measure the amount of water vapour in the air to find the **relative humidity**.

The relative humidity is the percentage of the maximum water the air could hold (at a specific temperature) that is actually in the air. So, a relative humidity of 51% would mean that the air contained 51% of the water it could hold at that temperature. This how to calculate relative humidity:

$$\text{Relative humidity} = \frac{\text{Grams of water per cubic metre of air}}{\text{Most grams of water per cubic metre the air can hold}} \times 100$$

Here are some relative humidity data for different world cities, recorded on a day in October.

Use the formula to fill in the gaps in the table.

City	Temperature (°c)	Relative humidity (%)	Water Vapour (g/m³)	Maximum Vapour (g/m³)
Cape Town	21		9.18	18
Dubai	28		2.75	25
London	7		4.06	7.8
Moscow	7	90		7.8
Mumbai	36	72		42
New Orleans	17	35	5.08	
Rio de Janeiro	23	92	18.86	
Rome	21	86		18
Timbuktu	38	23	11.04	

Calculating the Weather

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Core worksheet

When investigating the weather, it is often useful to compare data to long term averages. This gives meteorologists a sense of how weather conditions over a small period, often a month, relate to larger climate and weather patterns. Percentages are particularly helpful for making such comparisons, especially when investigating **rainfall** and **hours of sunshine**. There are two different ways to do this: calculate the percentage anomaly or calculate the percentage difference.

1. Calculate the percentage anomaly

This means that you turn your information into a percentage of the average.
So, if 79.8mm of rainfall fell one January and the average January rainfall was 95mm the anomaly would be 79.8 ÷ 95 which is 84% as a percentage.

The table below contains information from the Met Office about hours of sunshine each month in the UK in 2006. **Calculate the sunshine percentage anomaly for each month.**

2. Calculate the percentage difference

This means that you turn the difference between your information and the average into a percentage of the average. The method is:

$$\text{Percentage difference} = \frac{\text{New value} - \text{average}}{\text{Average}} \times 100$$

A **negative** answer shows there has been a percentage **decrease** and a **positive** answer shows there has been a percentage **increase**. Using the table, **find the percentage difference of sunshine for each month.**

Compare your percentage anomalies and percentage differences. What do you notice?

Month	Sunshine [hours] 2006	Sunshine [hours] 1961-90 average
January	45	43.9
February	67.9	63.1
March	89	100.6
April	164.2	141.9
May	179.6	179
June	210.3	175.9
July	253.3	167.1
August	137.9	158.1
September	140.1	120.2
October	86.6	88.5
November	78.6	58.3
December	43.3	39.3

Super Big Standard Form

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Starter worksheet

Standard form is a way of writing really big and really small numbers to make them easier to use. From astronomers comparing distances between galaxies to microbiologists measuring viruses, standard form is used all over the world. Scientists, in particular, find it very helpful.

Remember that standard form looks like this:

5.2×10^6

A power of 10

Number between 1 and 10

So, 5.2×10^6 means $5.2 \times 1\ 000\ 000 = 5\ 200\ 000$.

Pluto is approximately 3 600 000 000 miles away from the Sun. In standard form,

$$3\ 600\ 000\ 000 = 3.6 \times 1\ 000\ 000\ 000 = \mathbf{3.6} \times \mathbf{10^9}$$

The power is **9** because $1\ 000\ 000\ 000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$, which is 10 multiplied by itself **9** times.

Activity:

Don't think about this for long - just take a few seconds to guess:

how old are you … in seconds?

Compare your answer with the person sitting next to you.

(With a partner?) **Try and work out the real answer**. (You'll need a calculator)

Bear in mind…
there are 60 seconds in a minute, 60 minutes in an hour, 24 hours in a day and 365.25 days in year.

Your answer should be pretty big. You could tell your friends your exact age, down to the last second, but your answer will only be accurate for one second. Instead, it is more helpful to round your answer to **3 significant figures**.

Your answer will still be a bit of a mouthful so it is even more helpful to write it in standard form. **Have a go at expressing your answer in standard form.**

Now tell your friends how old you are!

Super Big Standard Form

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multiplying opportunities

Core worksheet

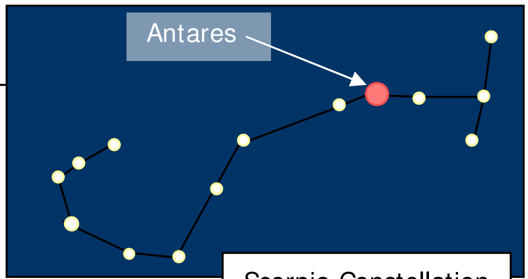
Sometimes you might want to expand standard form and return a number to its ordinary form.

For example, the average distance of Saturn from the sun is 1.43×10^9 km.

1.43×10^9 = $1.43 \times 1\,000\,000\,000$ = **1 430 000 000** km

Can you undo all the standard form in this article?

Rewrite the article with all the numbers in bold type written out in full.

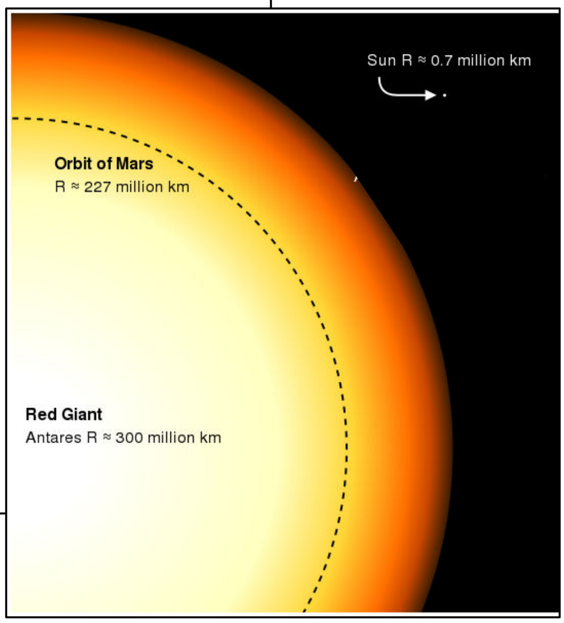


Scorpio Constellation

The brightest star in the Scorpio constellation is a red supergiant star called Antares. Even though Antares and our solar system are both in the Milky Way galaxy, Antares is **4.94×10^{15}** km away from the Earth. Astronomers estimate that Antares is just one of roughly **2.5×10^{11}** stars in the Milky Way.

The radius of Antares is **3.0×10^{8}** km – that's wider than the orbit of Mars and roughly 430 times larger the Sun's radius. Have a look at the scaled picture to see how they compare. At this scale, the Earth is too small to see because its radius is 109 times smaller than the sun's radius. **1.3×10^{6}** Earths would occupy the same volume as the Sun!

In about **5×10^{9}** years the Sun will become a red giant star and its radius will be at least **1.4×10^{8}** km. That's 200 times the length it is now and big enough to swallow up the inner planets – Mercury, Venus, Earth and Mars.



Super Big Standard Form

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Core worksheet

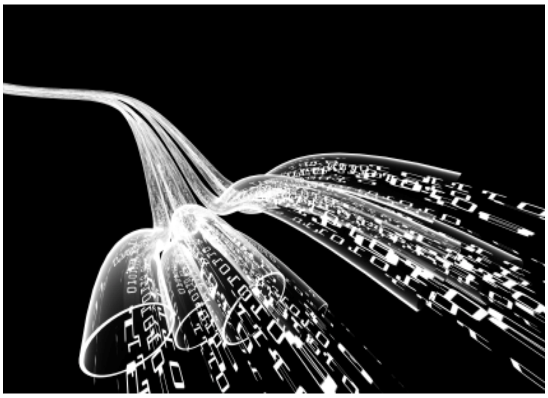
Calculations using standard form

What is $(1.5 \times 10^6) \times (4 \times 10^8)$?

Multiply the first part of each number together: **1.5 x 4**.

Then multiply the second part of each number together: $10^6 \times 10^8$. This is 10 multiplied by itself 6 and then 8 times, 14 times altogether, so $10^6 \times 10^8 = 10^{14}$.

So the answer is $1.5 \times 4 \times 10^6 \times 10^8 = 6 \times 10^{6+8}$
$= 6 \times 10^{14}$



1. What is $(5 \times 10^7) \times (4.2 \times 10^3)$? Express your answer in standard form.

The Milky Way is the galaxy of stars that we live in. It is approximately 6.76×10^{11} times longer than the diameter of the sun. The sun's diameter is 1.4×10^6 km.

Because distances in space are so much greater than distances on earth, astronomers have created special units to use in their measurements. One of these units is called a **light year**, which is the distance that light travels in a year. For measuring really massive distances, astronomers use another unit, called a **megaparsec**.

One light year is 9.46×10^{15}m and there are 3.26×10^6 light years in one megaparsec.

Read the text box and answer these questions:

2. What is the length of the Milky Way? Express your answer in standard form.

3. How many metres are there in a megaparsec? Express your answer in standard form.

We can divide two numbers in standard form by applying division to the first parts and to the second parts separately, then multiplying the results

$$(9 \times 10^{10}) \div (5 \times 10^7) = (9 \div 5) \times (10^{10} \div 10^7) = 1.8 \times 10^3$$

Try writing these numbers out in full to see why this works.

4. What is $(2.1 \times 10^{15}) \div (8.4 \times 10^8)$? Express your answer in standard form.

5. The brightest star in the night sky is Sirius and it is 8.14×10^{16} m from the earth. How many light years away is Sirius?

Super Big Standard Form

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multiplying opportunities

Advanced worksheet

Addition and subtraction

What is **(2.5×10^4) + (7×10^6)** ?

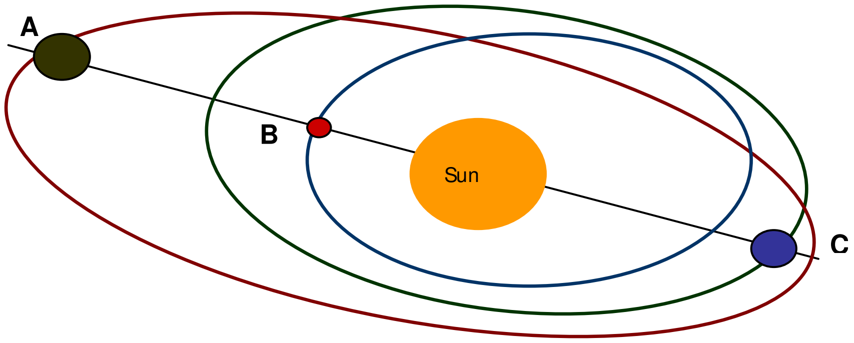
You could do this by turning both back into normal numbers, adding them and then putting the answer in standard form. Or you could try **factorising** the numbers as shown below.

$$(2.5 \times 10^4) + (7 \times 10^6) = (2.5 \times 10^4) + (7 \times 10^2 \times 10^4)$$
$$= (2.5 + [7 \times 10^2]) \times 10^4$$
$$= (2.5 + 700) \times 10^4$$
$$= 702.5 \times 10^4$$

You can use the same method for subtraction.

1. (a) Write 702.5×10^4 in standard form.
(b) What is (8.1×10^9) + (5.6×10^5)?
(c) What is (6.8×10^{11}) – (4×10^{10})?

For a short period of time, 3 planets and the sun they orbit all lie on the same straight line:



The distance from the centre of planet B to centre of the sun is 5.76×10^6km
The distance from the centre of planet B to the centre of planet C is 2.33×10^8km
The distance from the centre of planet A to the centre of planet C is 3.107×10^9km

2. Use this information to find:

The distance of planet A from the sun.
The distance of planet C from the sun.
The distance between planet A and planet B.

Super Small Standard Form

more maths grads
multiplying opportunities

Starter worksheet

Standard form is a way of writing really big and really small numbers to make them easier to use. From astronomers comparing distances between galaxies to microbiologists measuring viruses, standard form is used all over the world. Scientists, in particular, find it very helpful.

For numbers smaller than **1**, standard form looks like this:

7.1×10^{-4}

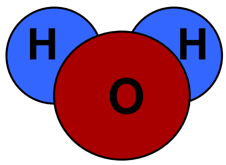
Number between 1 and 10 (7.1)

A negative power of 10 (10^{-4})

So, 7.1×10^{-4} means 7.1 x 1/10 000 = 7.1 x 0.00001 = 0.000071

Water is made up of lots of tiny molecules. Each water molecule is made of one **oxygen** atom (**O**) and two **hydrogen** atoms (**H**) … **H_20**.

The diameter of an oxygen atom is roughly 0.00000014 mm
= 1.4 x 0.0000001
= **1.4×10^{-7}** mm



The diameter of a hydrogen atom is roughly 0.000000074 mm.
What is this in standard form?

Match the number to its standard form!

0.0035

0.0000000000000000035

0.000000000000000000035

0.0000035

0.0000000000000035

0.000000000000000035

3.5×10^{-3} | 3.5×10^{-5}
3.5×10^{-6} | 3.5×10^{-15}
3.5×10^{-16} | 3.5×10^{-17}
3.5×10^{-18} | 3.5×10^{-19}
3.5×10^{-20} | 3.5×10^{-25}

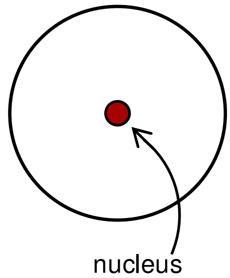
Super Small Standard Form

Core worksheet

Sometimes you may need to expand standard form and return a number to its ordinary form.

Take a closer look at an oxygen atom: scientists have discovered that at the centre of every atom is a tiny **nucleus**. A cloud of really really tiny particles called **electrons** circle around this nucleus.

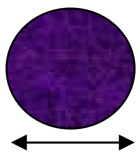
The nucleus of an oxygen atom has a radius of roughly 3.15×10^{-12} mm.



$$
\begin{aligned}
\mathbf{3.15 \times 10^{-15}} &= 3.15 \times 1 / 10^{12} \\
&= 3.15 \times 1 / 1\ 000\ 000\ 000\ 000 \\
&= 3.15 \times 0.000000000001 \\
&= \mathbf{0.00000000000315} \text{ mm}
\end{aligned}
$$

Have a go yourself…

Expand the standard form and rewrite these five lengths as normal numbers:

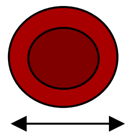


2.8×10^{-8} m

Echovirus

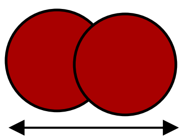
Width: 8×10^{-5} m

Average human hair



7×10^{-6} m

Red Blood Cell



1.21×10^{-10}

Oxygen molecule (O_2)

2×10^{-6} m

E. Coli bacterium

Super Small Standard Form

more maths grads
multiplying opportunities

Advanced worksheet

Calculations using standard form

What is $(3.2 \times 10^{-5}) \times (4 \times 10^{-7})$?

We can use the same method we used for Super Big standard form:

Multiply the first part of each number together … **3.2×4** … and multiply the second part of each number together … **$10^{-5} \times 10^{-7}$**

So, the answer is **$3.2 \times 4 \times 10^{-5} \times 10^{-7}$** $= 12.8 \times 10^{-5+(-7)}$

$= 12.8 \times 10^{-12}$

$= 1.28 \times 10^{-13}$

Division follows the same ideas as multiplication:

$(3.2 \times 10^{-5}) \div (2 \times 10^{-11}) = (3.2 \div 2) \times (10^{-5} \div 10^{-11})$

$= 1.8 \times 10^{-5-(-11)}$

$= 1.8 \times 10^{6}$

(notice how big this is compared to the numbers being divided!)

Let's take a closer look at the nucleus of our oxygen atom.

The nucleus is actually made from particles called **nucleons**. There are two types of nucleons: **protons** and **neutrons**. The number of **protons** in the nucleus tells you the name of the atom – every oxygen atom contains **8** protons, every hydrogen atom contains **1** proton.

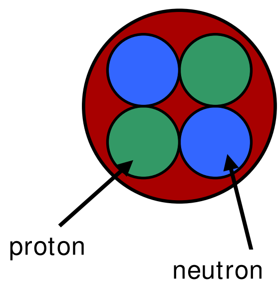
The mass of one nucleon is approximately **1.66×10^{-24}** g.

Oxygen atoms contain 8 protons and 8 neutrons. Hydrogen atoms contain 1 proton and no neutrons. Remember that a water molecule has two hydrogen atoms and one oxygen atom.

What is the total number of nucleons in one water molecule?
What is the approximate mass of one water molecule?

In one drop of water there are 1.4×10^{21} water molecules.
What is the approximate mass of a drop of water?

Did you notice that we ignored the electrons when we calculated the mass of water? The mass of one electron is roughly 1800 times smaller than the mass of one nucleon.
What is the mass of one electron?



Every atom contains the same number of electrons as protons.
What is the total number of electrons in one water molecule?
What is the approximate mass of all the electrons in one drop of water?

Circle Geometry and Pi

π = 3.14159265358979323846264…

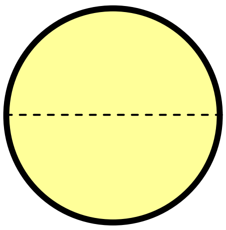
Starter worksheet

What's the point of π? It's a number, a little bigger than 3.14, that helps us relate how wide a circle is to its area or circumference.

It helps people:

- work out the volume of a tin of soup – and so make more profit selling it
- calculate the amount of steel needed to make a pipe
- decide how much paint is needed to mark the lines on a sports playing field
- put together x-ray data to pinpoint exactly where someone's leg is broken
- program better graphics for a video game
- design ways to pack more songs into an iPod

…and lots more!



The number π is just a number, like 2.5 or -3. It was called π - a Greek letter that we pronounce like **pie** – because it's the first letter of the Greek word for perimeter. Given a circle, the distance around its perimeter (measuring along the heavy black line) is known as its **circumference**. The distance straight across the circle (measuring along the dashed line) is its **diameter**.

When a circle's diameter is 1 metre, its circumference is 3.14159… metres.

What makes π important is that for **any** circle, if you divide the length of the circumference by its diameter, you get this same number. The letter π is just used so that we don't spend all our time writing out 3.14159…

Thousands of years ago, engineers and designers needed to know how to work out volumes and areas, but they didn't know exactly what value of π to use. Even these days, sometimes you only need to know that the circumference is roughly **three** times the diameter. In most business applications using a value of 3.14 is fine. But in medical or electronic applications, you have to be much more accurate, perhaps using a value of 3.1416 or 3.141593.

Read the information above and use it to answer the following questions.
The key thing you have to decide is what value of π you are going to use.

1. If a circle has diameter 10 metres, how long is its circumference to the nearest metre?

2. If a circle has diameter 100 metres, how long is its circumference to the nearest metre?

3. Philip Bell is an engineer. He wants to estimate the circumference of a circular room which has diameter 20 metres. He needs to know the circumference to the nearest metre. What is it, and what approximate value for π could he use and still get the right answer?

4. Farzana Iqbal is a medical student. She needs to calculate the circumference of a circular red blood cell which has diameter 691.2 nanometres, to the nearest nanometre. What is it, and which approximate values for π could she use and still get the right answer?
(A nanometre is a millionth of a millimetre.)

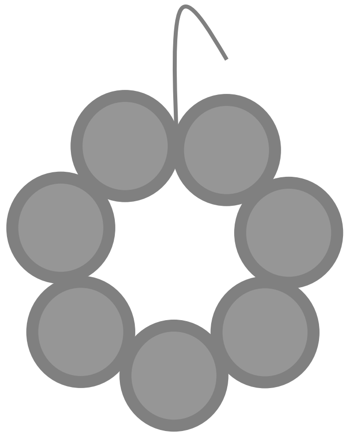
Circle Geometry and Pi

more maths grads
multiplying opportunities

π = 3.14159265358979323846264...

Core worksheet

Melissa Chen is a fashion design student. She decides to design some earrings like the one shown in the picture, with seven silver circles all of the same size.

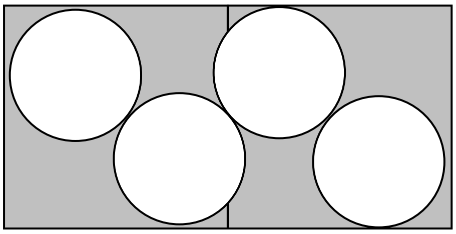


Melissa wants to make a pair of these earrings but doesn't want to spend more than £30 on materials. She looks up her online supplier and finds out the following prices.

A plain sterling silver hook costs £0.80.
A 50mm by 50mm sheet of 0.55mm thick silver costs £11.26.
A 50mm by 50mm sheet of 1.1 millimetre thick silver costs £22.51.
A 50mm by 100mm sheet of 0.55mm thick silver also costs £22.51.
A 50mm by 100mm sheet of 1.1 millimetre thick silver costs £45.02.

1. Melissa first decides to see how big she can make the earrings if she uses 0.55mm thick silver. She needs to fit 14 circles of the same size on a sheet of silver 50mm by 100mm.

   Investigate ways of packing 14 circles on the sheet. It may help you to use some scrap paper to draw the sheet and different ways of packing circles on it. For example, the diagram below shows how you can pack four circles of diameter about 30mm on a 50mm by 100mm sheet.



2. When Melissa explains her plan to her teacher, he suggests that if she used the thicker silver, she would be able to re-use any leftover silver by soldering it together. From a 50mm by 50mm sheet of 1.1mm thick silver, how big can Melissa make the circles for her earrings now?

   In this situation she doesn't have to think about placing circles on the sheet, but just the total area of silver sheet involved.

Surface area and volume

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multiplying opportunities

Starter worksheet

There are many industries where you need to work out the surface area or volume of an object. If a company manufactures steel pipes for a factory, they need to know how much steel they'll need to buy and will have to calculate the surface area of the pipes.

Most companies have to store materials in a warehouse. They need to know how much warehouse space they'll need and so will have to estimate the volume of the materials they need to store. The company can't ask for too little warehouse space but if they ask for a lot more space than they need, they'll have to pay the cost for that space and so reduce their final profits.

Circumference C of a circle with radius r	$C = 2 \pi r$
Area A of a circle with radius r	$A = \pi r^2$
Volume V of a cylinder with radius r and height h	$V = \pi r^2 h$

Using the formulas given in the table above, answer the questions below. Remember that the diameter of a circle is twice its radius. The number π is 3.14 to 2 decimal places.

1. What is the area of a circle of radius 6cm?

2. If a circle has circumference 44cm, what is its radius to the nearest centimetre?

3. If a cylindrical steel pipe has radius 60cm and length 200cm, what is its surface area? (You will have to think about the formula for this one. Draw a picture!)

4. What is the surface area in square metres of a cylindrical steel pipe which has radius 40cm and length 5 metres? (You'll need to convert the units to metres.)

5. What is the volume in cubic metres of a cylindrical steel drum which has radius 50cm and height 1 metre?

6. What would the formula be for the total surface area of a closed cylindrical steel drum with radius r and height h? (A closed drum has a curved surface, a top and a bottom.)

7. What is the total surface area in square **metres** of a closed cylindrical steel drum with radius 30cm and height 80cm?

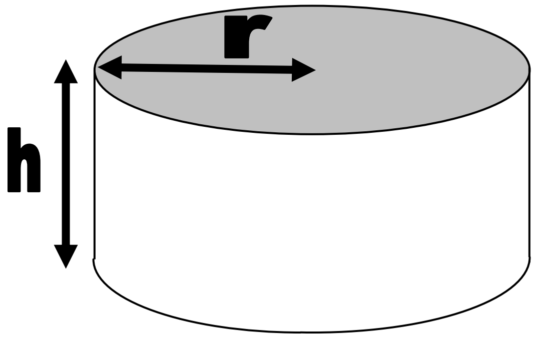
Surface area and volume

more maths grads
multiplying opportunities

Core worksheet

You work for a company called Kitchips that makes nutritious cat food. They are bringing out a new product and you will have to decide how the product should be sold and at what price.

Remember to convert all units first!



Basic Mix costs £200 per cubic metre.
Superior Mix costs £250 per cubic metre.
Top Quality Mix costs £280 per cubic metre.

Aluminium costs £15 per square metre.

You have to work out the cost of producing a tin of cat food. Suppose that the radius **r** of a tin is 6cm and its height **h** is 7cm.

You need to add two costs together for each tin: the aluminium needed to make the tin and the cost of filling the tin with one of the mixes. Use the information above.

	Basic Mix	Superior Mix	Top Quality Mix
Cost of making the tin			
Cost of filling the tin with mix			
Total cost			

Surface area and volume

Advanced worksheet

You work for a company called Kitchips that makes nutritious cat food. They are bringing out a new product and you will have to decide how the product should be sold and at what price.

Basic Mix costs £200 per cubic metre.
Superior Mix costs £250 per cubic metre.
Top Quality Mix costs £280 per cubic metre.

Aluminium costs £15 per square metre.

Results of the Marketing Survey

Kitchips commissioned a survey to find out what sizes of tin people would buy. The conclusions were:

The volume of the tin should be between 400cm^3 and 600cm^3.
The ratio of diameter to height should be between 1.5 and 1.7.
The tin should be between 8cm and 16cm in diameter.
The tin should be between 4cm and 10cm in height.

Find a size of tin satisfying the conclusions of the survey. To make things easier for your production staff, the radius and height should each be a whole number of centimetres.

Height of tin	
Radius of tin	

Use your work from previous sheets to help you complete the table below, using the size of tin you just worked out.

Total cost of tin of Basic Mix	
Total cost of tin of Superior Mix	
Total cost of tin of Top Quality Mix	

Speed and Distance Graphs

more maths grads
multiplying opportunities

Starter worksheet

This worksheet is for use with the racing car animation in this resource.

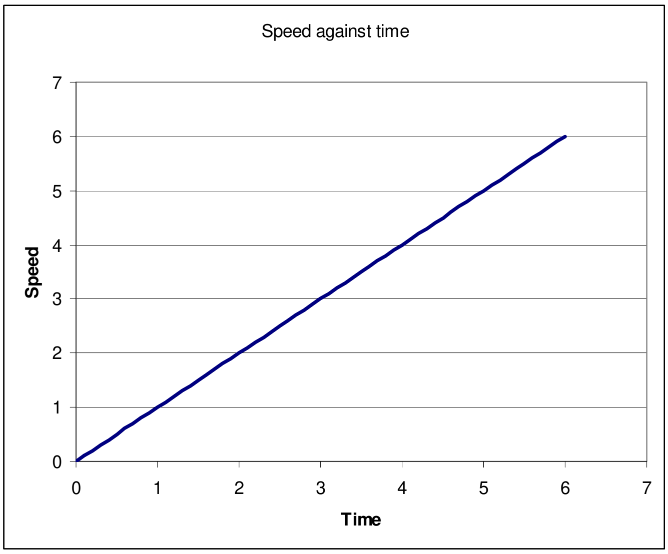
In the animation, set the car to start stationary (0 mph or 0 m/s). We will measure the distance travelled by the car and plot it in a graph against time. At zero time, the distance travelled is zero.

Acceleration measures how much speed changes over time. In a car there's a pedal called the accelerator which you push with your foot to change the acceleration. Push it right to the floor and you will give the car a high acceleration. This means that the car's speed will get faster and faster. When driving a normal car, it's hard to get it to go extremely fast because of the friction of the car's tyres rubbing against the ground and the force of the wind pushing back on the car. Racing cars are designed so that the air goes smoothly over them so that the car can go faster. In our animation we ignore these effects, but when designing a real car they're very important.

We can measure distance in metres and speed in metres per second. Acceleration measures how speed changes over time, so we measure it in units of **metres per second per second**. We can write "metres per second" as m/s and "metres per second per second as m/s^2.

If you have a constant acceleration of 1 metre per second per second that means that the speed of the car gets faster by 1 metre per second every second. Starting at time zero with a speed of 0 metres per second, we get the following table of values.

Time	0 seconds	1 second	2 seconds	3 seconds	4 seconds	5 seconds	6 seconds
Acceleration	1 m/s^2	1 m/s^2	1 m/s^2	1 m/s^2	1 m/s^2	1 m/s^2	1 m/s^2
Speed	**0 m/s**	**1 m/s**	**2 m/s**	**3 m/s**	**4 m/s**	**5 m/s**	**6 m/s**



But a car's speed doesn't suddenly jump from 0 m/s to 1 m/s without going through all the values in between. In fact, the graph of speed against time looks like this – it's a straight line.

Answer the following questions:

1. With a constant acceleration of 1 m/s^2 as shown in the table and graph, what's the speed of the car at the following times? (a) 0 seconds; (b) 4 seconds; (c) 2.5 seconds; (d) 8 seconds; (e) 0.5 seconds?

2. What do you notice?

Speed and Distance Graphs

more maths grads
multiplying opportunities

Core worksheet

Now let's think about the distance the car travels. You will be able to use the animation to help your understanding. Again we let the acceleration be 1 m/s^2.

How far does the car travel in the first six seconds? Guess an answer.

Here are three facts. See if you can convince yourself that they must be true.

Fact 1
The car's speed is 0 metres per second at the start and increases to 6 metre per second at the end. The speed goes steadily through all the values between 0 m/s and 6 m/s as shown in the graph on the previous worksheet.

Fact 2
Since the car is moving by the end, it must have moved some distance, so the answer is definitely bigger than 0 metres.

Fact 3
Since the car never goes faster than 6 metres per second, and it only travels for 6 seconds, the car cannot possibly have travelled more than 36 metres.

1. Use the animation. How far does the car actually travel in the first six seconds?

2. Use the animation to help you complete the table. What do you notice?

Time	0 seconds	1 second	2 seconds	3 seconds	4 seconds	5 seconds	6 seconds
Acceleration	1 m/s^2	1 m/s^2	1 m/s^2	1 m/s^2	1 m/s^2	1 m/s^2	1 m/s^2
Speed	0 m/s	1 m/s	2 m/s	3 m/s	4 m/s	5 m/s	6 m/s
Distance	0 metres						

3. Discuss what's going on with a partner or in a group. Here are some questions to explore.

What happens if you try different values of acceleration?
What happens if you try different starting speeds?

Averages

Starter worksheet

Moving averages can be used whenever data has been recorded over a period of time. They are useful because they smooth out up and downs in a set of data to show the underlying **trend**.

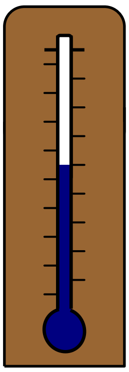
To find a **3 point moving average** in a set of data you take the mean of the first, second and third values, then the mean of the second, third and fourth values, then the mean of the third, fourth and fifth values... and so on. You continue to take the mean of groups of three, moving along one value each time, until you reach the end of the data.

1. How do you think you would calculate a 5 point moving average?

The table on the left below contains information from the UK Met Office and shows the mean temperature in the UK each July from 1998.

2. Complete the second table with the three year moving averages over this period of time.

Year	**Mean July Temperature** [°C]
1998	14
1999	15.9
2000	14.1
2001	15.3
2002	14.4
2003	16.2
2004	14.5
2005	15.3
2006	17.8
2007	14.3
2008	15.3



Years	**Mean July Temperature** [°C]
1998-2000	(14+ 15.9+ 14.1) ÷ 3 = 14.7
1999-2001	
2000-2002	
2001-	

Use graph paper. Draw a pair of axes and mark “Years” on the x-axis, “Temperature” on the y-axis.

3. On these axes, plot a graph of the first table and a graph of the second table.

(Think carefully about where you plot the points corresponding to the second table!)

4. Do you think that the data supports the statement that UK summers are getting warmer?

Averages

more maths grads
multiplying opportunities

Core worksheet

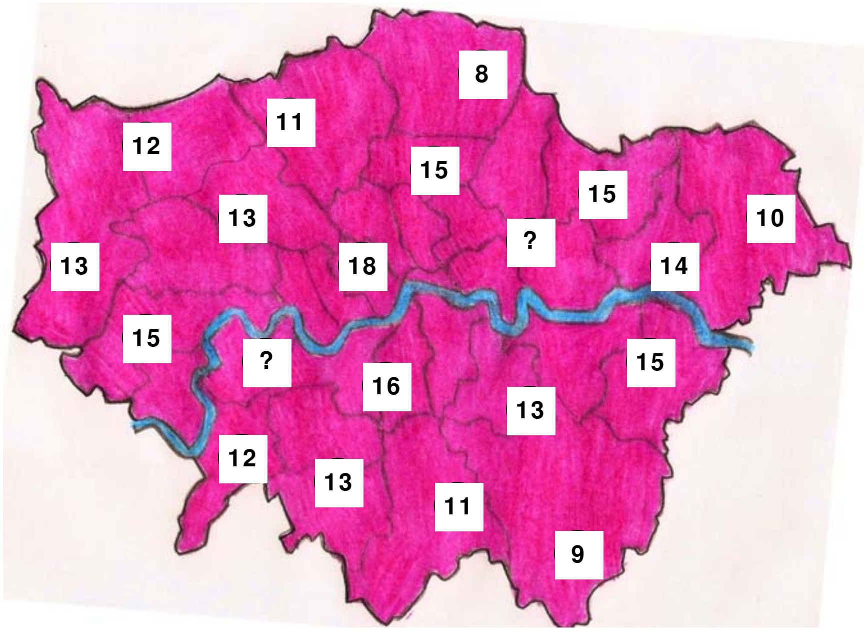
One morning the temperature of different areas of London was measured (in °C). This map shows the results.

Unfortunately, two of the temperatures became smudged. Can you work out what they are, using this information?

The **modal** temperature is 15°C.

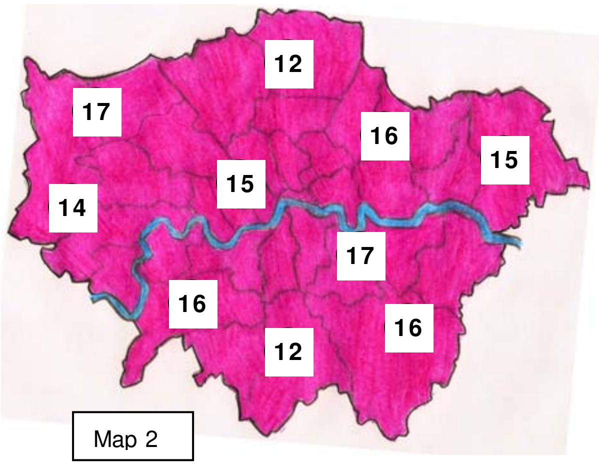
The **mean** temperature is 13°C.

1. What are the two missing temperatures?



Below, are two more temperature maps from the same week. The three maps show the temperatures in London on Thursday, Friday and Saturday. Use the information below to work out which day each map represents.

- The median temperature on Friday was lower than the modal temperature on Thursday.
- On Saturday, the median temperature was two higher than the modal temperature.



Map 2

Map 3

2. Which day does each map represent?

3. The temperature in London was recorded over a different week. The mean temperature for the whole week was 22°C. The mean temperature at the weekend was 17°C. What was the mean temperature over the other five days of the week?

Probability and Law

Starter worksheet

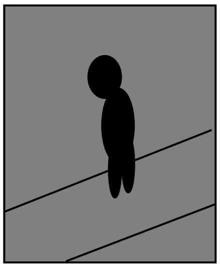
If a legal case comes to trial in a British court, there may be two or more different arguments made as to what happened. Then those listening have to decide how likely each argument is to be correct.
Here we look at a concrete example.

Suppose that Mr Robb is accused of smashing the window of a jewellery shop in a small town and stealing several items of valuable jewellery. The prosecution puts together all the evidence they have that Mr Robb was the person who did this. Mr Robb also has a legal team whose job is to defend him and argue that he didn't do it. The prosecution has one main source of evidence: some video footage from the CCTV (closed circuit television) camera at the corner of the street.

It was dark at the time so the CCTV footage doesn't show much detail. It shows the figure of a person walking down the street towards the jewellery shop, smashing the window, disappearing from view and then walking quickly away from the shop.

A CCTV expert says that the person in the footage is definitely male. She also says that the probability of the person being between 6 foot 1 inch and 6 foot 4 inches tall is 0.95. This means that there's a 95% chance that the person shown has a height in this range and a 5% chance that their height is outside this range.

Mr Robb is 6 foot 3 inches tall and was known to be in the area at the time because he was seen outside a pub on the same street earlier in the evening.



1. What is the probability that the man in the CCTV footage has a height which is either less than 6 foot 1 inch or more than 6 foot 4 inches?

2. Suppose that there are 10000 people in the town and that roughly half of them are male. Suppose that 15% of the men have a height between 6 foot 1 inch and 6 foot 4 inches. How many people in the town are men with a height in this range?

3. What is the probability that a randomly chosen person who lives in the town is a man with a height in the range 6 foot 1 inch to 6 foot 4 inches?

4. Suppose that there were about 200 people who were seen in the part of town near the jewellery shop that evening. What is the probability that a randomly chosen person who lives in the town was seen in that part of town that evening?

5. Assume that the probability of being seen in that part of town is **independent** of your height and whether you're male or female. Approximately how many people seen in that part of town were men with a height in the range 6 foot 1 inch to 6 foot 4 inches?

(Hint: Because you are assuming that the probabilities are independent, you can multiply together the probability of being seen in that part of town with the probability of being a man whose height lies in the range given. Then multiply the answer by the number of people living in the town.)

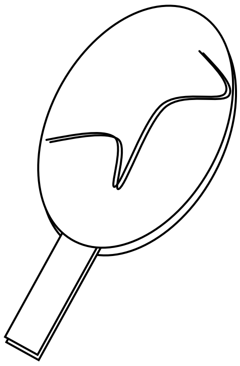
6. Do you think that it's reasonable to make this assumption that the probabilities are independent?

7. Do you think that the prosecution needs to find more evidence against Mr Robb? Why?

Probability and Law

Core worksheet

On the starter sheet we looked at the case of Mr Robb and a particular item of CCTV evidence. Mr Robb was accused of breaking into a jewellery shop. The closed circuit television evidence supported this accusation but not strongly. However the prosecution also found some **DNA evidence** at the scene of the crime. This was collected from some dried blood on a shard of broken glass.



DNA (deoxyribonucleic acid) is a molecule found in all human and animal cells. Although all of the cells in one person's body will have the same DNA molecules in them, different people usually have different DNA molecules. Identical twins have the same DNA! Your DNA will have some parts in common with your mother's DNA and with your father's DNA. This is one way in which personality traits and the way you look can get passed on to you from your parents. When a forensic scientist looks at DNA evidence, they have special techniques to analyze it and get the most information out of it.

The police forensic scientist analyzed the blood and compared the DNA to the national police database. Mr Robb's DNA was not on the database. However the blood sample DNA showed a strong match to that of his older sister Mrs Steel, who had previously been convicted of shoplifting. The scientist was able to show that the blood came from a male.

Based on this strong match to his sister's DNA and on the supporting evidence from eyewitnesses and the CCTV camera, the police arrested Mr Robb, and took a DNA sample from him.

1. The forensic scientist explains that because there was only a small amount of blood to test, he could only use some of the less accurate DNA tests available. Out of the national database of ten million people, five thousand people had DNA that matched the sample, including Mr Robb. Suppose that there are sixty million people in the country. Approximately how many of these sixty million people would you expect to have DNA matching the sample?

2. What is the probability that a person chosen randomly from the population of the entire country has DNA matching the blood sample?

3. There were around 200 people who were in the right part of the town at the time when the jewellery store was broken into. The statistical expert in the court suggests that about 40 of them would be male and of height between 6 foot 1 inch and 6 foot 4 inches, as suggested by the CCTV evidence. What is the probability that a person chosen randomly from the population of the entire country was one of these 40 men?

4. Assuming that the probabilities of having DNA that matches the blood sample and being one of the 40 men in the area and of a suitable height are **independent**, what is the probability of a person chosen randomly from the population of the entire country satisfying both these conditions?

5. Do you think that it's reasonable to make this assumption that the probabilities are independent?

6. Do you think that the prosecution needs to find more evidence against Mr Robb?

7. Is there any other information that would be particularly useful when assessing the DNA evidence?

Presenting Data

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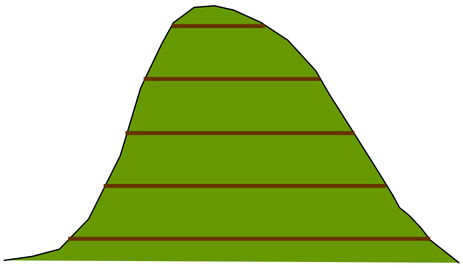
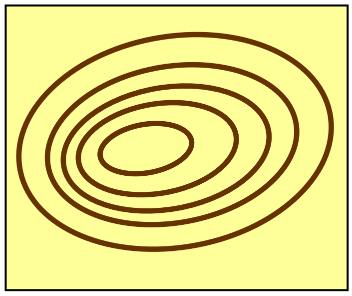
Starter worksheet

Cartographers (map makers) show the physical shape of the land by drawing **contour lines**. A contour line joins points that are all the same height above sea level.

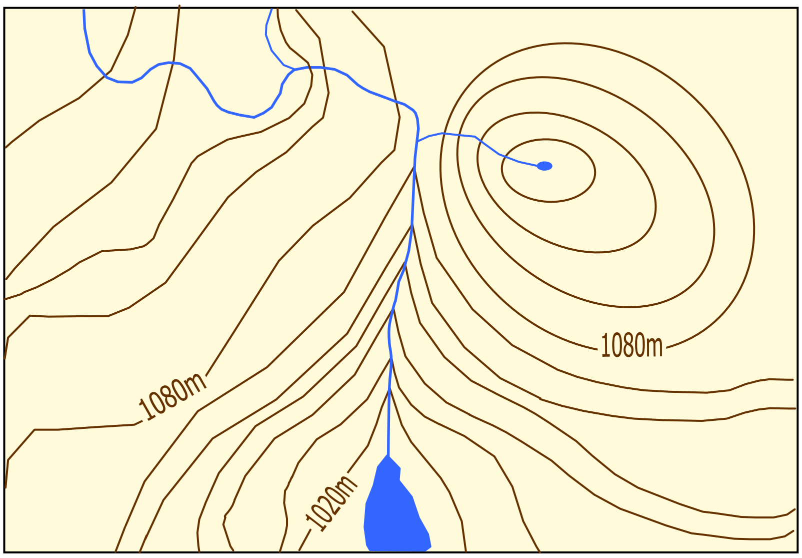
The map on the left represents the hill on the right. Every 10m increase in height, an imaginary circle has been drawn around the hill and marked as a contour line on the map. The map shows the lines as they would appear from above – if you looked down at the hill from an aeroplane.





Some of the contour lines are close together and some are more spread out. This is because some parts of the hill have steeper slopes than others.

Now try these questions about the map below:



1. Mark the 1050m contour line.
2. What is the height of the highest contour line? Mark it on the map.
3. The cartographer knows there is a farm at 1055m. On the map, mark one point the farm could be.
4. Developers are planning to build a new road in this area. The road must stay above 1078m and below 2000m. On the map, draw one route the road could take.

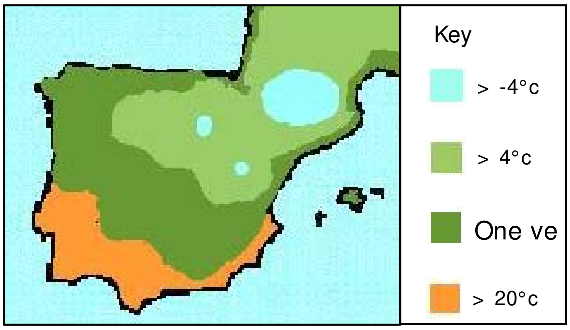
Presenting Data

Core worksheet

Meteorologists (people who study the weather) study patterns in the weather to make forecasts. By drawing lines on their **weather charts**, meteorologists can find and display weather patterns.

Newspapers often publish weather maps, like this one of Spain and Portugal. Lines called **isotherms** have been drawn on the map. Isotherms join points with the same temperature.

The areas between the lines have been coloured to show areas of similar temperature. This makes it very easy to see the rough temperature of any place.



Drawing isotherms

Drawing isotherms on a map can be very tricky. One way to make it easier is to draw in a triangular grid. If you know the temperatures at the corners of the triangles, you can find the isotherms.

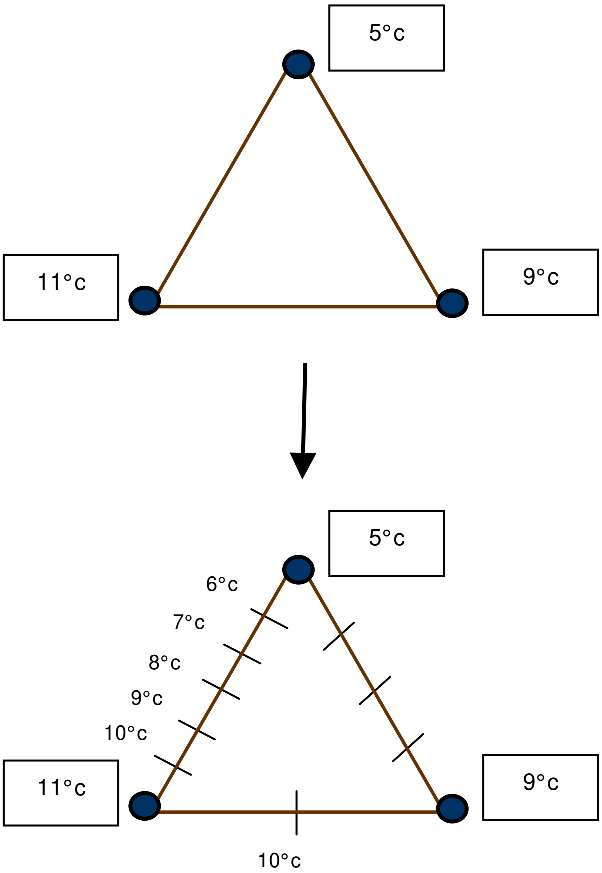
Look at this triangle. Somewhere on the line that joins 11°c and 9°c the temperature must be **10°c**. We can estimate that 10°c is halfway along the line.

We can use the same idea to mark the temperatures that lie on the other two lines – assuming that the temperatures are evenly spaced along the lines.

1. Finish marking in the temperatures on the bottom triangle.

To draw the isotherms, just draw lines to join all points that are at the same temperature. Two different isotherms never cross each other!

2. Draw the isotherms on the bottom triangle.



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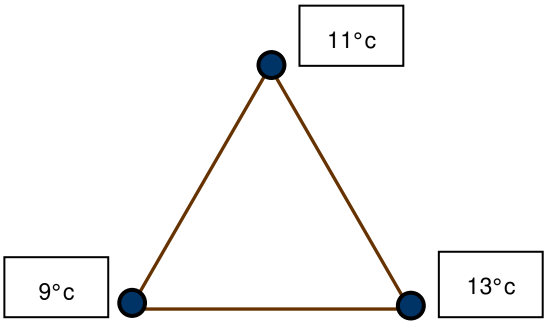
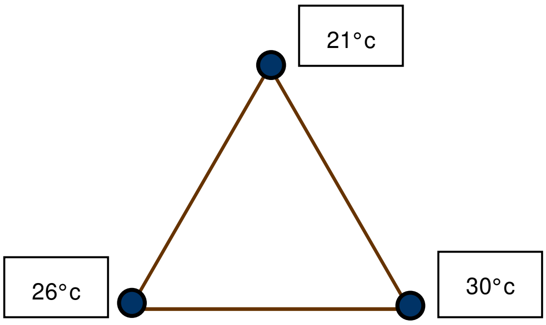
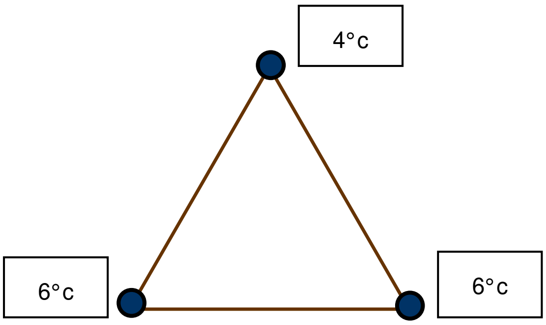
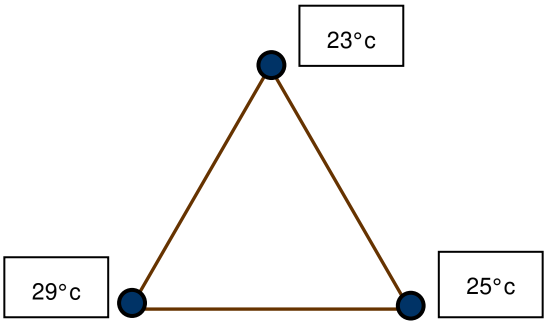
Core worksheet

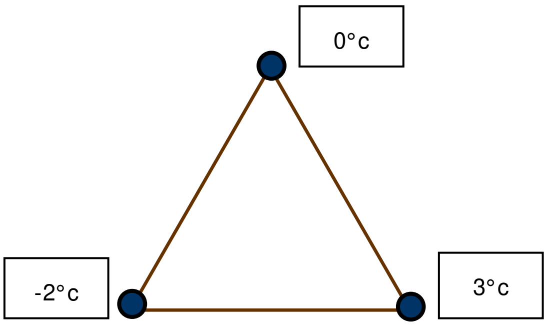
Using the information on the other sheet to help you, fill in the isotherms on each of the triangles shown.

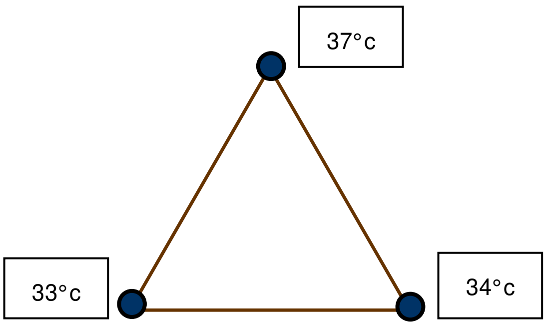












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Advanced worksheet

One very important thing that meteorologists study is **atmospheric pressure.** Atmospheric pressure is the force put on us by the weight of the air. It is measured in **millibars**.

Different weather conditions change atmospheric pressure. By displaying atmospheric pressure on weather charts, meteorologists can see these changes and forecast the weather.

Meteorologists draw lines to join points of the same pressure.

The lines are called **isobars** and they are marked every **4 millibars**.

Can you draw the isobars on this map of Europe?

You will need to draw the 988, 992, 996, 1000, 1004, 1008 and 1012 isobars. Remember: no isobar ever crosses any other isobar.

A triangular grid has been drawn to help you. There are pressures marked at the corners of this grid. You might find it helpful to find where each isobar crosses the grid lines.