

Enrichment **Activities**

How does maths apply to a card trick? Can you really use maths in fun and exciting ways? In fact, card trickery is just one area out of a huge number of entertaining and innovative ways that maths can be used.

This booklet has been designed to highlight the ways that maths is used in daily life. The puzzles guide you through from the question to the discussion of how much fun maths can be and to highlight the many ways in which it can be applied to an everyday setting.

This booklet has been produced by *more maths grads*. *more maths grads* (MMG) is a three-year project funded by the Higher Education Funding Council for England to develop, trial and evaluate means of increasing the number of students studying mathematics and encouraging participation from groups of learners who have not traditionally been well represented in higher education.

MMG has also been working with the Higher Education Funding Council for Wales to translate and disseminate its resources throughout Wales. This booklet is just one example of the translated resources.

Fractals Problem

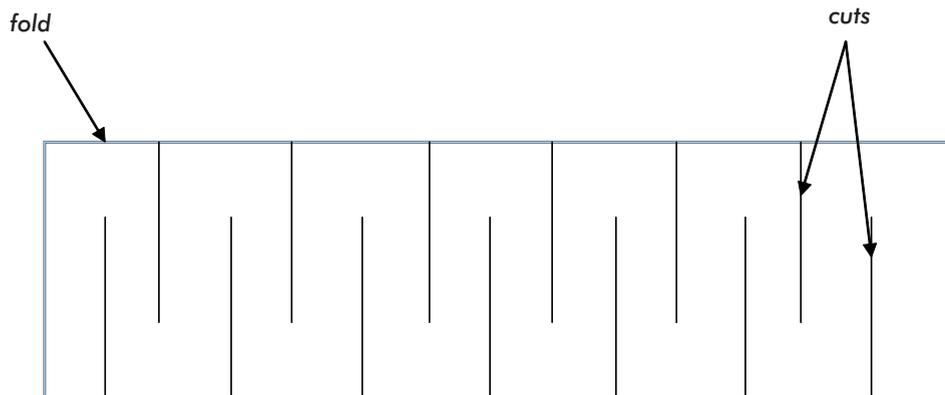
Equipment required:

- Lots of A4 paper (about 5 sheets per student)
- Scissors
- Pictures of fractals

Instructions:

- Give students a pair of scissors and a piece of paper.
- Ask them to make a hole in the paper so that they can put their hand through it. The remaining paper must form a complete loop.
- Give students a new piece of paper.
- Ask the students to make a hole so that they can put their head through. Again the remaining paper must form a complete loop.
- Give students a third piece of paper.
- Ask the students to make a hole so that they can put their whole body through. Again the remaining paper must form a complete loop.
- If students succeed, challenge them to get themselves and as many friends as possible through the paper.

(SOLUTION: The solution is to fold the paper in half and make cuts on alternate sides as shown in the diagram)



Discussion:

Following discussion could include:

- Changing the perimeter of a shape without changing the area
- Fractals and some images of fractals
- Similar ideas in nature, for example the surface area of the lungs and intestine

All the Aces

Arrange 16 playing cards in a chessboard formation. Choose a "line", either horizontal or vertical. Then "fold" along that line, as though you were folding a piece of paper.

Keep choosing lines to fold along until you have folded up the whole pack. You should find that whatever way you have folded up the pack, all the cards end up facing the same way. This trick relies on the principle that whenever you fold up a chessboard of cards (try different sizes), they will always end up facing the same way.



Now replace four of the upturned cards with Aces:



Now turn these aces over.

The idea is to prearrange the pack. Here this has been done with the Aces in the 3rd, 8th, 11th, and 16th positions. Then if you deal out the pack left to right, top to bottom, you can "randomly" turn over cards 1,6,9 and 14. When you fold up the pack, the Aces will be facing the "wrong" way.

You have "magically" turned the four upturned cards into Aces.

Get the pupils to think about why folding up a chess board will always end up with all the cards facing the same way.



The idea for this trick was given to us by Rob Eastaway. (www.robeastaway.com)



Johnny Ball's favourite Card Trick

1. Make sure you have a complete set of 52 cards without the jokers.
2. Deal out nine cards. Get a pupil to select a card from these nine and show it to the class, but make sure you don't see it.
3. Put their selected card on the top of the other eight cards. Put the nine cards on the bottom of the pack. As you can see, the special card is now in the 44th position.
4. Now start dealing out the pack, counting down from ten. You are waiting for the cards to "speak" to you. With younger pupils it is probably a good idea to make a bit of a show of this.



5. The cards "speak" if you deal out the number you have just spoken. For example if I have counted down to seven and deal the number seven at the same time, then the cards have "spoken" and you stop there.



6. Then start a new pile, again counting down from ten. If you get down to one and find that the cards haven't spoken, then cap the pile with the next card placed face down. (Meaning you have 11 cards in the pile.)
7. Continue doing this until you get four piles. You will most probably have some number cards facing upwards.



Ask the class what you could do with these numbers. (Hopefully after some directed prompting the class will suggest you add these numbers and maybe even that you deal out that number of cards from the remainder of the pack.) In the example, the upturned numbers add up to 17. Now deal out 17 cards and turn over the 17th card. Miraculously, this will be the card chosen at the beginning.

8. ****On the very rare occasion that the cards don't speak, turn over the card that you are capping the fourth pile with. This will be the special card.****

Explanation: Get the pupils to notice where you put the special card. (It is always in the 44th position.) This trick is all about a special way of counting to 44.

Imagine if no cards spoke. Then you would find that you would have four piles of eleven cards and the 44th card would cap the final pile.

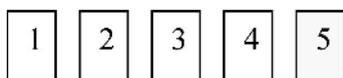
However if the cards do speak, then notice what happens. If the card speaks on "seven", then you have counted out four cards and are seven cards short of eleven. If the cards speak on "five", then you have counted out six cards and are five cards short of eleven. The upturned cards are telling you how many cards you need to make piles of eleven. So overall, if you add up the numbers on the upturned cards, it will tell you how many cards you need to count out in order to make 44 cards.

Mystic Richard

You need to have two people who have learnt the trick. The thinking behind it is quite complex, so we have only done this with the older age groups.

1. An audience member shuffles the pack and deals out five cards to person B.
2. Person B says confidently that they can “psychically” transmit the identity of one of the cards to “Mystic Richard” (Person A).

How Person B must arrange the cards.



- Any five cards must feature at least one suit more than once.
- Place the cards of repeating suit in Positions 2 and 5.
- Mystic Richard will have to guess card 5. By looking at card 2, they will immediately know the suit of card 5.
- Now cards 1, 3 and 4 will tell Mystic Richard how many places they need to count on from card 2 in order to get to card 5's place value. (For example if card 2 is the 3 of hearts and card 5 is the 6 of hearts, they must count on three places.)
- Card 2 and card 5 can be arranged so that you only ever need to add on a maximum of six to card 2 in order to get card 5. (Assuming the pack is cyclic, if you add three to the King of Hearts, this will give you the 3 of Hearts.)
- Now place cards 1, 3 and 4 in order, according to the table below. Assume Ace is low.

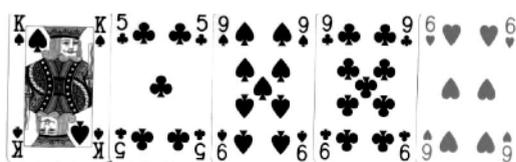
Order of cards 1, 2 and 3	How many you should count on from card 2, to get to card 5
Low Medium High	1
Low High Medium	2
Medium Low High	3
Medium High Low	4
High Low Medium	5
High Medium Low	6

- If cards 1, 3 and 4 contain a repeating value (e.g. two Kings) then you need to use Bridge Order: Clubs < Diamonds < Hearts < Spades.

3. Now Mystic Richard simply needs to look at the first four cards and they will be able to work out the identity of the fifth card (as long as person B has arranged them correctly!).

Example

Suppose you are dealt the cards below.



The Spades are repeated as well as the Clubs, so you have a choice. I would put the 5 of Clubs in the second position and the 9 of Clubs in the fifth position. This means I need to add four to the 5 of clubs in order to get the 9 of Clubs. I therefore need to put the other three cards in the order: medium high low. (Nine, King, Six)



Mystic Richard can now work out what the fifth card is.

Discussion with the class:

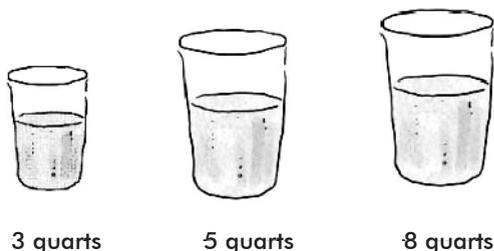
Some pupils will probably not believe you that this can be done for any combination of cards

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Three Jugs Problem

Two friends who have an eight-quart jug of water wish to share it evenly. They also have two empty jars, one holding five quarts, the other three. How can they each measure exactly four quarts of water?

Investigate!



Number Puzzles

Try these tricks on your friends and family.

Birthday trick

Get your friend to do the following steps.

1. Write down the month of their birth as a number. For example February is 2.
2. Double this number.
3. Add 5.
4. Multiply by 50.
5. Add their age.
6. Subtract 65.

At this point get your friend to call out the total.

In your head secretly add 115. You will now have a three- or four-digit number. For a three-digit number, the first digit will be the month of their birth. For a four-digit number, the first two digits will be the month of their birth. The last two digits will be their age. So for example: 240 would mean that they were born in February and that they are 40 years old.

A number trick which doesn't go out of date

Get your friend to do the following steps.

1. Write down the year in which they were born.
2. Write down an important year in their lifetime.
3. Write down the number of years which have elapsed since that important year.
4. Write down their age this year on their birthday.
5. Add up the numbers on their list.
6. Divide by 2.

They should be left with the current calendar year.

Maths at Work Mpegs

Exciting video resources are available on the Maths Careers website. The Mpegs aim to contextualise the link between maths and industry. These are also available with Welsh subtitles.

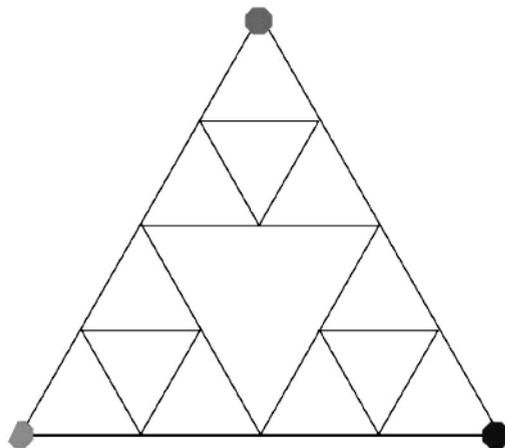
Please visit www.mathscareers.org.uk

Sierpinski Game

Play in pairs. Decide who will go first. You'll need a ruler and a pen or pencil. Player 1 chooses a corner to start at: red, green or blue. Player chooses one of the nine small triangles pointing upwards as the target.

A move consists of moving your counter exactly halfway to one of the corners. So the move "red" means move halfway from where you are to the red corner. Player 1 has to find a combination of moves which move from the starting corner into the target triangle. The player has to get *inside* the triangle, not on any of its edges. Count the number of moves it takes – that's Player 1's score. Now swap over. Keep a total of the scores and see who can get the lowest score over several rounds.

Extension: Divide each triangle up into smaller triangles. If you choose a triangle pointing upwards how many moves does the other player need to get inside it?

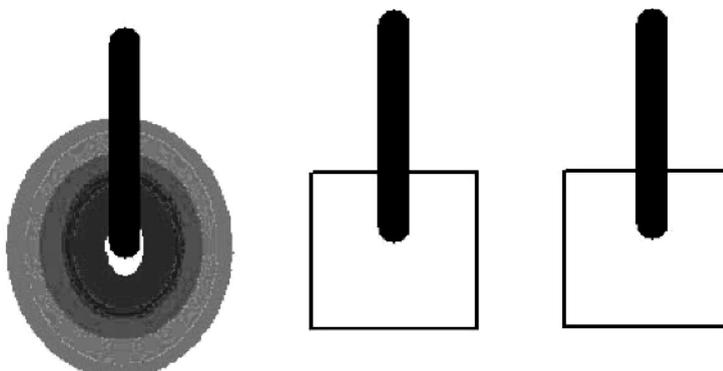


The Towers of Hanoi

Move the rings to the third tower so that they are arranged from biggest to smallest.

Rules:

1. You can only move one ring at a time to a neighbouring tower.
2. You cannot place a bigger ring on top of a smaller ring.
3. Try to do this in the least number of moves possible.



Which A level subject gives you access to a thousand different careers?

Actuary
 Accountant Aerodynamicist Aeronautical
 Engineer Agronomist Air Traffic Controller
 Aircraft Tracking Researcher Archaeologist Architect Archivist
 Army Officer Astronaut Astronomer Audio Software Engineer Auditor Bank
 Manager Biochemist Bioinformatician Biokineticist Biologist Biomathematician Biomedical
 Engineer Biostatistician Bookkeeper Botanist Building Surveyor Business
 Decision Analyst Business Owner Cartographer Chemical Engineer Chemist
 Chiropractor City Trader Civil Engineer Civil Servant Complexity Modeller Computer Games Developer
 Computer Programmer Computer Scientist Computer Tomography Technician Construction Manager Cosmologist
 Credit Risk Modelling Analyst Cryptologist Data Analyst Data Mining Specialist Database
 Developer Defence Analyst Dentist Detective Dietician Doctor Ecological
 Modeller Econometrician Economist Electronic Engineer
 Environmental Health Officer Epidemiologist Financial Analyst
 Financial and Investment Manager Financial Engineer Flight Engineer Fluid
 Dynamicist Food Scientist Forensic Scientist Geneticist Geographer Geologist
 GIS Analyst Hospital Manager Hydrologist Information Security Officer Insurance Analyst
 Insurance Underwriter Intelligence Specialist Investment Banker Investment Manager Lawyer
 Lecturer Lighting Director Logistics
 Manager Management Consultant
 Marine Biologist Maritime Sensor
 Researcher Market Researcher Mathematical
 Biologist Mathematical Modeller Mathematics
 Researcher Mechanical Engineer Medical
 Physicist Medical Sales Representative Medical
 Statistician Merchant Banker Merchant Navy Officer
 Metallurgist Meteorologist Metrologist
 Microbiologist Naval Defence Analyst
 Navigator Nuclear Scientist Numerical
 Analyst Occupational Therapist
 Oceanographer Oil Refinery Engineer
 Operations Researcher Optometrist
 Patent Attorney Pharmacist
 Pharmacologist Physicist
 Physiotherapist Pilot Policy Advisor
 Portfolio Analyst Project Manager

 Psychologist Quality Assurance Manager
 Quantitative Analyst Quantity Surveyor
 Racing Car Designer Radiographer RAF
 Officer Risk Analyst Rocket Scientist Royal
 Navy Officer Sales Forecaster Science
 Communicator Science Journalist Science
 Librarian Scientific Instrument Designer
 Software Engineer Sound Technician
 Sports Engineer Sports Scientist
 Statistician Stockbroker Stress
 Analyst Structural Engineer Surgeon
 Systems Analyst Teacher Technical Author
 Telecommunications and IT Analyst Town
 Planner Toxicologist Transport Planner

YOU
DO THE
MATHS!

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