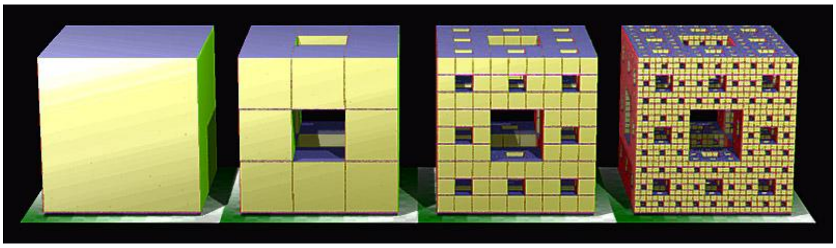
The Maths Inside...
THE MENGER SPONGE

Austrian mathematician Karl Menger first described what became known as the Menger Sponge in 1926. He was working in the field of **topology** and was trying to develop a definition of dimension. But the Menger Sponge is in fact an instance of what is nowadays termed a **fractal**.

The Sponge is constructed by dividing a unit cube into an array of 27 smaller cubes of side one third, then removing the central cube and six cubes at the centre of each face. This procedure is repeated with each of the 20 remaining smaller cubes.

At any point in the process, there will be 20^n cubes remaining where n is the number of iterations which have been carried out. The Sponge is the limit of this construction process as n tends to infinity.



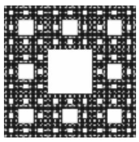
The first four iterations in creating a Menger Sponge

The Sponge is a key and somewhat surprising mathematical object. Menger was able to show that it had topological dimension only 1, which means that in many ways its behaviour is much closer to that of curves and line drawings than to surfaces or solid volumes.

At the same time, the Menger Sponge in fact contains a distorted copy of every other topologically 1-dimensional set. This means that any curve, line, diagram or graph can be distorted to fit in the sponge without self intersection and mathematicians describe the Sponge as a ‘universal curve’.

Georg Cantor described a linear version of the Sponge in 1883. Known as the ‘Cantor set’, this might be considered to be the prototype fractal. The planar analogue, first described by Wacław Sierpiński in 1916, is known as the ‘Sierpiński carpet’.

Construction of the Cantor set by repeatedly removing the middle third of intervals



Sierpiński carpet

Very roughly, fractals are geometric shapes that have irregularities at all scales, however small. For instance the Menger sponge has holes of arbitraily small sizes. Moreover, the Menger Sponge is a 'self-similar' fractal in that it is made up of 20 scaled copies of itself, each one scaled by a factor of one third.

Using traditional machining methods, it is very difficult to make a Menger Sponge. The research team show a menger sponge model fabricated by rapid prototyping method.

This document was produced by the Mathematics Promotion Unit
at the London Mathematical Society and Institute of Mathematics & its Applications
in conjunction with the From Music to Sand Painting project and the Royal Society

July 2007