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## Starter worksheet

If a legal case comes to trial in a British court, there may be two or more different arguments made as to what happened. Then those listening have to decide how likely each argument is to be correct.
Here we look at a concrete example.

Suppose that Mr Robb is accused of smashing the window of a jewellery shop in a small town and stealing several items of valuable jewellery. The prosecution puts together all the evidence they have that Mr Robb was the person who did this. Mr Robb also has a legal team whose job is to defend him and argue that he didn't do it. The prosecution has one main source of evidence: some video footage from the CCTV (closed circuit television) camera at the corner of the street.

It was dark at the time so the CCTV footage doesn't show much detail. It shows the figure of a person walking down the street towards the jewellery shop, smashing the window, disappearing from view and then walking quickly away from the shop.

A CCTV expert says that the person in the footage is definitely male. She also says that the probability of the person being between 6 foot 1 inch and 6 foot 4 inches tall is 0.95 . This means that there's a $95 \%$ chance that the person shown has a height in this range and a 5\% chance that their height is outside this range.

Mr Robb is 6 foot 3 inches tall and was known to be in the area at the time because he was seen outside a pub on the same street earlier in the evening.

1. What is the probability that the man in the CCTV footage has a height which is either less than 6 foot 1 inch or more than 6 foot 4 inches?
2. Suppose that there are 10000 people in the town and that roughly half of them are male. Suppose that $15 \%$ of the men have a height between 6 foot 1 inch and 6 foot 4 inches. How many people in the town are men with a height in this range?
3. What is the probability that a randomly chosen person who lives in the town is a
 man with a height in the range 6 foot 1 inch to 6 foot 4 inches?
4. Suppose that there were about 200 people who were seen in the part of town near the jewellery shop that evening. What is the probability that a randomly chosen person who lives in the town was seen in that part of town that evening?
5. Assume that the probability of being seen in that part of town is independent of your height and whether you're male or female. Approximately how many people seen in that part of town were men with a height in the range 6 foot 1 inch to 6 foot 4 inches?
(Hint: Because you are assuming that the probabilities are independent, you can multiply together the probability of being seen in that part of town with the probability of being a man whose height lies in the range given. Then multiply the answer by the number of people living in the town.)
6. Do you think that it's reasonable to make this assumption that the probabilities are independent?
7. Do you think that the prosecution needs to find more evidence against Mr Robb? Why?

On the starter sheet we looked at the case of Mr Robb and a particular item of CCTV evidence. Mr Robb was accused of breaking into a jewellery shop. The closed circuit television evidence supported this accusation but not strongly. However the prosecution also found some DNA evidence at the scene of the crime. This was collected from some dried blood on a shard of broken glass.


DNA (deoxyribonucleic acid) is a molecule found in all human and animal cells. Although all of the cells in one person's body will have the same DNA molecules in them, different people usually have different DNA molecules. Identical twins have the same DNA! Your DNA will have some parts in common with your mother's DNA and with your father's DNA. This is one way in which personality traits and the way you look can get passed on to you from your parents. When a forensic scientist looks at DNA evidence, they have special techniques to analyze it and get the most information out of it.

The police forensic scientist analyzed the blood and compared the DNA to the national police database. Mr Robb's DNA was not on the database. However the blood sample DNA showed a strong match to that of his older sister Mrs Steel, who had previously been convicted of shoplifting. The scientist was able to show that the blood came from a male.

Based on this strong match to his sister's DNA and on the supporting evidence from eyewitnesses and the CCTV camera, the police arrested Mr Robb, and took a DNA sample from him.

1. The forensic scientist explains that because there was only a small amount of blood to test, he could only use some of the less accurate DNA tests available. Out of the national database of ten million people, five thousand people had DNA that matched the sample, including Mr Robb. Suppose that there are sixty million people in the country. Approximately how many of these sixty million people would you expect to have DNA matching the sample?
2. What is the probability that a person chosen randomly from the population of the entire country has DNA matching the blood sample?
3. There were around 200 people who were in the right part of the town at the time when the jewellery store was broken into. The statistical expert in the court suggests that about 40 of them would be male and of height between 6 foot 1 inch and 6 foot 4 inches, as suggested by the CCTV evidence. What is the probability that a person chosen randomly from the population of the entire country was one of these 40 men?
4. Assuming that the probabilities of having DNA that matches the blood sample and being one of the 40 men in the area and of a suitable height are independent, what is the probability of a person chosen randomly from the population of the entire country satisfying both these conditions?
5. Do you think that it's reasonable to make this assumption that the probabilities are independent?
6. Do you think that the prosecution needs to find more evidence against Mr Robb?
7. Is there any other information that would be particularly useful when assessing the DNA evidence?
