## Teachers' Guide to Worksheets

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## List of Animations and Bonus Content

algebra.exe Beam animation linking to Algebraic Expressions topic
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## Teachers' Guide to Worksheets

## Coordinates / Scale Factors

These worksheets link into the video From Concept to Construction where two young structural engineers talk about their work. They mention driving piles and how important it is to lay a solid foundation for a building. You can see the piles in position in the foundations of a building shown in the first half of the 15 -minute film. Possible extensions of this would involve getting hold of some real blueprints for a building or looking at Google maps / Multimap and perhaps some joint work with your Geography department on maps and coordinates.

## Coordinates Starter Worksheet

The seven other positions for the piles are:

$$
(3,2),(1.5,4),(2.5,3.5),(3.5,3),(2,5),(3,4.5),(4,4) .
$$

You'd need 36 piles in total, so 27 more piles, at the following positions:
$(4,1.5),(5,1),(6,0.5),(4.5,2.5),(5.5,2),(6.5,1.5),(5,3.5),(6,3)$, $(7,2.5),(2.5,6),(3.5,5.5),(4.5,5),(5.5,4.5),(6.5,4),(7.5,3.5)$, $(3,7),(4,6.5),(5,6),(6,5.5),(7,5),(8,4.5),(3.5,8),(4.5,7.5)$, $(5.5,7),(6.5,6.5),(7.5,6),(8.5,5.5)$.

## Coordinates Core Worksheet

1. (Check that marks match up to given coordinates.)
2. There are nine piles on land, ten in shallow water, seven in deep water and four in very deep water, giving a total of $£ 2000 \times 9+£ 3500 \times 10+£ 4500 \times 7+£ 5500 \times 4=£ 105000$
3. The other position needs eight piles on land, five in shallow water, seven in deep water and ten in very deep water, giving a total cost of $£ 120$ 000 . So this would have been $£ 13500$ more expensive.

## Scale Factors Starter Worksheet

1. The wheel of the model car has diameter 1.875 cm .
2. The steering wheel of the model car has diameter 1.25 cm .
3. 2 square metres is 20000 square centimetres.
4. Dividing 20000 by 576 (which is $24 \times 24$ ) gives 34.72 square cm .

## Scale Factors Core Worksheet

1. The scale of the map is approximately $1: 6250000$.
2. The coastline is approximately 2 metres ( 200 cm ) long on the screen.
3. Approximately 400000 pixels are needed to display the map. The map is 280 square centimetres in area, which represents 1.09 million square kilometres. Therefore each pixel represents approximately 3 square kilometres of area.

## Teachers' Guide to Worksheets

## Angles

Not every class can work with a football theme without half the students getting distracted and the other half getting bored. However this theme may prove useful for some classes, or for a collaboration with the PE or science department.

There are several ideas here: measuring and estimating distances; speed, distance and time; trigonometry; some simple physics; drawing scale diagrams; data handling. The worksheets lay out one way of working through these ideas but there must be countless variations. You could play a clip of a goal from youtube and ask students to estimate times and distances. The worksheets on quadratic equations in sport cover a related theme also seen in the TDA's recent advertising campaign.

The book "Beating the Odds" by Rob Eastaway and John Haigh covers much of the hidden mathematics involved in sport, from the arrangement of numbers on a dart boards to why footballs are the shape they are (truncated icosahedra!).

## Angles Starter Worksheet

1. The angle is 18.45 degrees - so 18 degrees to the nearest degree (but 19 degrees might also be reasonable, depending on the method).
2. (a) It will never reach the goal.
(b) It will take about 0.578 seconds (assuming no friction).

## Angles Core Worksheet

1. The angle is 11.9 degrees.
2. It will hit the line (goalpost) FG below G.
3. You need to kick the ball at a larger angle than 11.9 degrees.

## Angles Advanced Worksheet

The height y to 2 decimal places is:

- -0.64 metres for 5 degrees
- 0.35 metres for 10 degrees
- 1.34 metres for 15 degrees
- 2.35 metres for 20 degrees
- 3.40 metres for 25 degrees
- 4.49 metres for 30 degrees

Calculating y for more angles results in a height y of 2.46 metres for an angle of 20.5 degrees and 2.56 metres for an angle of 21 degrees, so the perfect angle to hit the top of the goalpost would be 20 degrees to the nearest degree. This ignores other factors such as wind speed / direction or the spin a player puts on the ball when they kick it.

## Teachers' Guide to Worksheets

## Surveying with Trigonometry

This topic is based around the idea of having a fixed collection of starting data and needing to get out a distance or angle as an answer. As for the worksheets on coordinates and scale factors, it can be better to bring this to life through concrete functional applications - perhaps involving colleagues working in the science, history or geography departments. Our examples are a bit bland: these days people often use GPS systems to navigate and computers to aid design, and the need for simple trigonometrical calculations has lessened.

Trigonometrical functions $\sin (x)$ are crucial to advanced mathematics at A-level and beyond, but the stage where students see sin, cos and tan purely in terms of calculations for triangles can be quite a difficult one to motivate.

If you have good ideas for how to make this aspect of trigonometry - or any other maths topic! - come to life, post them on the website of the National Centre for Excellence in the Teaching of Mathematics (www.ncetm.org.uk) on their communities forums. You can apply to them for small grants of money to help with projects designed to enhance mathematics teaching.

## Surveying with Trigonometry Starter Worksheet

1. $\mathrm{d}=\mathrm{c} /(1 / \tan \mathrm{A}+1 / \tan \mathrm{B})$
2. $\sin A=d / b$ and $s o b=d / \sin A$
3. $\sin B=d / a$ and $s o a=d / \sin B$
4. $d=2.17 \mathrm{~km}, \mathrm{a}=2.5 \mathrm{~km}, \mathrm{~b}=4.33 \mathrm{~km}$
5. $\mathrm{d}=1.24 \mathrm{~km}, \mathrm{a}=1.29 \mathrm{~km}, \mathrm{~b}=2.94 \mathrm{~km}$
6. $\quad d=4.77 \mathrm{~km}, \mathrm{a}=5.27 \mathrm{~km}, \mathrm{~b}=6.75 \mathrm{~km}$

## Surveying with Trigonometry Core Worksheet

1. $\quad a=(c \sin A) / \sin (180-(A+B))$
2. $\quad b=(c \sin B) / \sin (180-(A+B))$
3. $a=2.5 \mathrm{~km}, \mathrm{~b}=4.33 \mathrm{~km}$ (same as Q4 on starter worksheet)
4. $\mathrm{a}=1.29 \mathrm{~km}, \mathrm{~b}=2.94 \mathrm{~km}$ (same as Q5 on starter worksheet)
5. $a=5.27 \mathrm{~km}, \mathrm{~b}=6.75 \mathrm{~km}$ (same as Q6 on starter worksheet)
6. $a=3.01 \mathrm{~km}, \mathrm{~b}=1.78 \mathrm{~km}$
7. $a=3.82 \mathrm{~km}, \mathrm{~b}=1.21 \mathrm{~km}$
8. Rule $B$ is correct: $\sin (180-(A+B))=\sin (A+B)$
9. $a=(c \sin A) / \sin (A+B) ; b=(c \sin B) / \sin (A+B)$

## Teachers' Guide to Worksheets

## Sine Waves in Music

There's a lot you can do with a maths and music theme, these worksheets are here to give an idea of the maths involved in frequencies and in designing and tuning a keyboard instrument like a piano or electronic keyboard.

Access to keyboards, tuning forks or other instruments may be useful. There's scope for a joint project with the music department. See the "My Music" resource in the Bowland Maths set, freely available on DVD.

Images can be found online which match up the frequencies and notation with a picture of a keyboard: type "piano keyboard frequencies" into Google to see a selection. There are many websites which suggest further practical music / physics / mathematics projects, such as www.sciencebuddies.org.

## Sine Waves in Music Starter Worksheet

1. 2.00 , calculated from 880.00 divided by 440.00 .
2. Any pair of notes twelve spaces away from each other in the list: for example F2\# and F3\# or C5 and C6.
3. $1.059454545 \ldots$, or 1.059 to three decimal places.
4. Any notes next to each other in the list give approximately 1.059: for example F2 and F2\# or C5\# and D5.
5. 2093.00 - it should be double the frequency of C6.
6. $\quad 1108.72$ (or between 1108 and 1110 depending on rounding error).
7. A7, since 3520 is four times the frequency of A5.

## Sine Waves in Music Core Worksheet

1. $\quad$ C4 and C5
2. G4 (392.00 compared to 392.44)
3. F4 (349.23 compared to 348.84 )
4. A4 ( 440.00 compared to 436.05 - so not especially close)
5. $1.059435 .$. is not close to such a fraction: $18 / 17$ is possibly the best.

## Sine Waves in Music Advanced Worksheet

The questions here continue the theme from the previous sheets, asking the same question but in different ways.

1. The exact ratio will depend on the notes: it's approximately 1.059 .
2. All pairs of consecutive notes have roughly this ratio.
3. To one decimal place you get 2.0 if you raise 1.059 to the power 12.
4. Keyboard makers solve the problem by averaging out frequencies: the frequency of each note is a constant multiple of the previous note, chosen so the note an octave higher has exactly double the frequency. Every other note only has an approximate frequency.

## Teachers' Guide to Worksheets

## Algebraic Expressions

These worksheets fit with the second half of the film From Concept to Construction. The young structural engineers describe how a floor deflects in a building and that they have to make sure that it stays within sensible safety margins. They use straightforward GCSE maths algebra to do this - often working by hand to make estimates as it's faster than using computer software.

There's a Flash animation (beamlite.swf) which shows a weight sitting on a beam. Like all the Flash animations on this DVD it can usually be played through an Internet Explorer browser as well as by the flash player software on the DVD. You can adjust the length of the beam and the weight and get an idea of how much the beam deflects in each case.

## Algebraic Expressions Starter Worksheet

| 1. | $D=0.008 \mathrm{~m}(8$ millimetres $)$ | 2. | $D=0.04 \mathrm{~m}$ (4 centimetres) |
| :--- | :--- | :--- | :--- |
| 3. | $D=0.135 \mathrm{~m}(13.5$ centimetres $)$ | 4. | $D=0.405 \mathrm{~m}$ ( 40.5 centimetres) |

The beam bends more if the weight is heavier or if the beam is longer. Doubling the length of the beam makes much more difference than doubling the weight - indeed it increases the deflection by a factor of eight.

## Algebraic Expressions Core Worksheet

1. Taking $\mathrm{W}=1500 \mathrm{~kg}, \mathrm{D}=0.002 \mathrm{~m}, \mathrm{~L}=6 \mathrm{~m}$ and $\mathrm{k}=0.0001$ we can calculate $\mathrm{T}^{3}$ as 0.018 so $\mathrm{T}=0.27 \mathrm{~m}$ ( 27 centimetres) would be safe.
2. Taking $\mathrm{W}=1500 \mathrm{~kg}, \mathrm{D}=0.002 \mathrm{~m}, \mathrm{~L}=4 \mathrm{~m}$ and $\mathrm{k}=0.0001$ we can calculate $\mathrm{T}^{3}$ as 0.0053 so $\mathrm{T}=0.18 \mathrm{~m}$ (18 centimetres) would be safe.
3. Taking $\mathrm{W}=1500 \mathrm{~kg}, \mathrm{D}=0.002 \mathrm{~m}, \mathrm{~L}=3 \mathrm{~m}$ and $\mathrm{k}=0.0002$ we can calculate that $T$ is not safe at 0.13 m and needs to be 0.17 m ( 2 dp ).
4. Taking $\mathrm{W}=1500 \mathrm{~kg}, \mathrm{D}=0.002 \mathrm{~m}, \mathrm{~L}=3.5 \mathrm{~m}$ and $\mathrm{k}=0.0002$ we can calculate that $T$ is safe at 0.23 m thick -0.20 m would be safe.
5. Taking $\mathrm{W}=1500 \mathrm{~kg}, \mathrm{D}=0.002 \mathrm{~m}, \mathrm{~L}=5 \mathrm{~m}$ and $\mathrm{k}=0.0002$ we can calculate that $T$ is not safe at 0.25 m and needs to be 0.28 m ( 2 dp ).

## Algebraic Expressions Advanced Worksheet

Students' answers will vary but here's one possible argument.
If a person needs 50 cm of personal space then the maximum on one beam is 12 people, a total weight of 1500 kg . With $W=1500 \mathrm{~kg}, \mathrm{D}=0.003 \mathrm{~m}, \mathrm{~L}=6 \mathrm{~m}$ and $k=0.0002$ we can calculate that the thickness $T$ needs to be at least 29 cm . With the safety margin this goes up to 44 cm . The cost of a 44 cm thick beam is $£ 920$ at Anderson's, $£ 830$ at Boonville's and $£ 880$ at Clark and Carr. So the cheapest price for the floor is $£ 8300$ by buying ten beams at Boonville's.

## Teachers' Guide to Worksheets

## Simultaneous Equations

There are many different ways to teach simultaneous equations. The worksheets here are more to sketch out possible ways of approaching the subject. Simultaneous equations can be really hard to motivate because the simple problems you encounter don't need the formal mathematical algorithms to solve them.

The way that simultaneous equations are relied on in practice usually involves matrix mathematics and computers solving systems which have thousands or millions of sparse equations - sparse means there are lots of zero coefficients. What we've sketched in the core worksheet is a possible process for explaining a version of the Google Page Rank algorithm in a way that involves simultaneous equations. It investigates how a search engine might rank four webpages for popularity - and so involves a set of four simultaneous equations. It's hard to balance making the situation feel oversimplified and obtaining a system of equations which is possible to solve.

If you have good ideas for how to make simultaneous equations - or any other maths topic! - come to life, post them on the website of the National Centre for Excellence in the Teaching of Mathematics (www.ncetm.org.uk) on their communities forums. You can apply to them for small grants of money to help with projects designed to enhance mathematics teaching.

## Simultaneous Equations Starter Sheet

1. With simple problems like this there are lots of possible methods.
2. A banana costs 25 p and a mango $£ 1.14$.

## Simultaneous Equations Core Sheet

1. Two links point to $A$; two links to $B$; three links to $C$; one link points to $D$. Thus webpage C has the most links to it.
2. The grid should read (left to right): $0101 / 0011 / 1101 / 0100$.
3. Here's one way to solve the equations. The fourth row tells us that $b=3 d$. Putting everything in terms of $d$, we get: $a=1.333 \mathrm{~d}, \mathrm{~b}=3 \mathrm{~d}, \mathrm{c}=2.667 \mathrm{~d}$. Since $a+b+c+d=1$, this implies $(1.333+3+2.667+1) d=1$, and so $8 \mathrm{~d}=1$ and $\mathrm{d}=0.125$.
Therefore $a=0.167, b=0.375, c=0.333$ and $d=0.125$. So the webpages would be ordered as B, C, A then D.

I think most students would find the paper worksheet as it is currently presented too cluttered and that the problem looks much harder than it is. However it might help you design an advanced functional maths project based on these ideas which avoids going into any maths beyond GCSE level.

## Teachers' Guide to Worksheets

## Quadratic Equations in Sport

As mentioned in the notes on the Angles worksheets, not every class can work with a sports theme without half the students getting distracted and the other half getting bored. However this theme may prove useful for some classes, or for a collaboration with the PE or science department.

## Quadratic Equations in Sport Starter Worksheet

1. If $a=0$ then the graph is a straight line.
2. (a) $a=-1, c=10$
(b) $y=10$ is the maximum
(c) $y=-15$ is the minimum in that range
(d) Sketch should have clear axes and correct parabolic shape.
3. We need $c=0$ so that $y=0$ when $x=0$.

## Quadratic Equations in Sport Core Worksheet

1. $x=2 w^{2}(\cos \theta)^{2} \tan \theta / g-$ which $\operatorname{simplifies~to~} 2 w^{2} \sin \theta \cos \theta / g$.
2. (a) $y=-0.049 x^{2}+0.087 x$
(b) $y=-0.065 x^{2}+0.577 x$
(c) $y=-0.098 x^{2}+x$
(d) $y=-0.196 x^{2}+1.732 x$
(e) $y=-6.451 x^{2}+11.430 x$
3. (a) 1.77 metres
(b) 8.84 metres
(c) 10.20 metres
(d) 8.84 metres
(e) 1.77 metres
4. Throwing or kicking the ball at 45 degrees allows it to go furthest.
5. You could get the ball to go further by throwing or kicking it harder, which would increase its speed and increase the distance it goes.

## Teachers' Guide to Worksheets

## Business Equations

These worksheets can be used to support group working, particularly since it's easy to make it a competition between teams as to who can make the most profit. The context can easily be changed to involve the sale of different goods.

## Business Equations Starter Worksheet

The largest possible profit for each type of tin can be made as follows:
Selling the basic design tin at $£ 1.04$ gives 62 p profit per tin.
The number of people buying it is $50000-300 \times 104=18800$.
The total profit is $£ 11,656$.
Selling the standard design tin at $£ 1.50$ gives $£ 1$ profit per tin.
The number of people buying it is $50000-200 \times 150=20000$.
The total profit is $£ 20,000$.
Selling the distinctive design tin at $£ 2.01$ gives $£ 1.33$ profit per tin.
The number of people buying it is $50000-150 \times 201=19850$.
The total profit is $£ 26,401$.
The easiest way to find these answers is to use a spreadsheet and try lots of values - the graph of total profit against price is a quadratic graph with a maximum as described above. Algebraically, the problem is to maximize a function such as (p-42)(50000-300p), which is why the graph is quadratic.

## Business Equations Core Worksheet

This is the same as the starter worksheet but the equations are a little bit more complicated. This time the problem is to maximize a function that is of the form $(p-0.50)\left(30000 / p^{3}\right)$, and note that this time $p$ is in pounds not pence. The form of equation used means that the optimal price is always 1.5 times the cost. The largest possible profit for each type of tine can be made as follows:

Selling the basic mix at $£ 0.75$ gives $£ 0.25$ profit per tin. The number of people buying it is $30000 / 0.75^{3}=71111$. The total profit is $£ 17,778$.

Selling the superior mix at $£ 0.87$ gives $£ 0.29$ profit per tin.
The number of people buying it is $40000 / 0.87^{3}=60744$.
The total profit is $£ 17,616$.
Selling the top quality mix at $£ 0.99$ gives $£ 0.33$ profit per tin.
The number of people buying it is $50000 / 0.99^{3}=51531$.
The total profit is $£ 17,005$.

## Teachers' Guide to Worksheets

## Calculating the Weather

Meteorological contexts provide a good source of situations involving percentages. Weather anomalies in particular are interesting because they provide a situation where professionals have to deal with percentage values which exceed $100 \%$.

The 15-minute film Weather Forecasting included on this DVD shows several weather forecasters explaining how their job involves maths. This worksheet and two of the others (Averages, Presenting Data) provide possible tasks to pair with the film.

## Calculating the Weather Starter Worksheet

From top to bottom in the table: 51, 11, 52, 7.02, 30.24, 14.5, 20.5, 15.48, 48.
In this table the temperature is irrelevant to the calculation but provides useful context: the maximum vapour in the air is higher if the temperature is higher.

## Calculating the Weather Core Worksheet

| Month | Sunshine <br> Percentage Anomaly | Sunshine <br> Percentage Difference |
| :---: | :---: | :---: |
| January | $102.5 \%$ | $2.5 \%$ |
| February | $107.6 \%$ | $7.6 \%$ |
| March | $88.5 \%$ | $-11.5 \%$ |
| April | $115.7 \%$ | $15.7 \%$ |
| May | $100.3 \%$ | $0.3 \%$ |
| June | $119.6 \%$ | $19.6 \%$ |
| July | $151.6 \%$ | $51.6 \%$ |
| August | $87.2 \%$ | $-12.8 \%$ |
| September | $116.6 \%$ | $16.6 \%$ |
| October | $97.9 \%$ | $-2.1 \%$ |
| November | $134.8 \%$ | $34.8 \%$ |
| December | $110.2 \%$ | $10.2 \%$ |

The percentage difference records the value of the anomaly minus $100 \%$.

## Teachers' Guide to Worksheets

## Super Big or Small Standard Form

The film Powers of Ten (not on this DVD) can be found on the internet and is one way to get students thinking about both the sizes involved in the universe and also the standard form notation used to represent them.

Standard form is used a lot in science so it may be helpful to work with the science department to motivate the physics or biology behind the applications. The Bowland Maths "You reckon?" resource provides a different take on this topic.

## Super Big Standard Form Starter Worksheet

Activity: how old are you... in seconds? - this is a well-known starter exercise. On a person's fifteenth birthday they are 473 million, or $4.73 \times 10^{8}$ seconds old.

## Super Big Standard Form Core Worksheet 1

Undoing the standard form gives, respectively: 4940000000000 000, $250000000000,300000000,1300000,5000000000,140000000$.

## Super Big Standard Form Core Worksheet 2

1. $\quad 2.1 \times 10^{11}$
2. $3.08 \times 10^{22}$ metres
3. $\quad 9.46 \times 10^{17} \mathrm{~km}$
4. $2.5 \times 10^{6}$
5. 8.60 light years

## Super Big Standard Form Advanced Worksheet

1. 

(a) $7.025 \times 10^{6}$ (b) $8.09944 \times 10^{9}$
(c) $6.4 \times 10^{11}$
2. Distance of planet $A$ from the sun:
$2.88 \times 10^{9} \mathrm{~km}$
Distance of planet $C$ from the sun:
$2.27 \times 10^{8} \mathrm{~km}$
Distance between planets A and B : $2.87 \times 10^{9} \mathrm{~km}$

## Super Small Standard Form Starter and Core Worksheets

Hydrogen atom: $7.4 \times 10^{-8} \mathrm{~mm}$ (or $7.4 \times 10^{-11} \mathrm{~m}$ )
Echovirus: $0.000000028 \mathrm{~m} \quad$ Human hair: 0.00008 m
Red Blood Cell: 0.000007 m Oxygen molecule: 0.000000000121 m
E Coli: 0.000002 m

## Super Small Standard Form Advanced Worksheet

Number of nucleons: 18 (water molecules have 1 oxygen, 2 hydrogen atoms)
Approximate mass of a water molecule: $2.99 \times 10^{-23}$ grams
Approximate mass of a drop of water: 0.042 grams
Mass of one electron: $9.22 \times 10^{-28}$ grams
Total number of electrons in one water molecule: 10 (one for each proton)
Approximate mass of all electrons in one drop of water: $1.29 \times 10^{-5} \mathrm{grams}$

## Teachers' Guide to Worksheets

## Circle Geometry and Pi

Students often find dealing with $\pi$ difficult. To understand it, it helps to have some discussion about when it can be approximated by which number: 3, 3.14, 3.1416 and so on. In many practical situations (and in ancient history) approximating $\pi$ by 3 is good enough. Fifty years ago and before calculators were widely used, the approximation $22 / 7=3.1428571$... was used - and many people believed that this was the accurate value of $\pi$ as a result of this.

These days $\pi$ is also thought of as the value you get when you press the [ $\pi$ ] button on a calculator. The first worksheet is intended to support this kind of discussion and so familiarize students with the idea of what values can be used to approximate $\pi$.

There are several medical contexts which involve circle geometry: opticians need to understand the shape of an eye; radiographers use X-ray or CT scanner machines that need to construct three-dimensional images from twodimensional slices. Technology constantly changes but the maths is often straightforward trigonometry or geometry.

The investigation in the second worksheet is more about circle geometry than about $\pi$. Advanced students might also be encouraged to think about threedimensional packings of spheres.

Several resources online currently support projects like this. The Dimensions videos may be of interest for some geometrical topics: http://www.dimensionsmath.org/Dim E.htm.

## Circle Geometry and Pi Starter Worksheet

1. $\quad 31$ metres (you need to use a value for $\pi$ at least as accurate as 3.14)
2. $\quad 314$ metres (you need to use a value for $\pi$ at least as accurate as 3.142)
3. 63 metres (he needs to use 3.14 or more accurate values)
4. 2171 nanometres (she needs to use 3.1416 or more accurate values)

## Circle Geometry and Pi Core Worksheet

1. The investigation is partly to see how circles can pack and partly to see how large you can make 14 circles of the same size that will fit on this sheet. It's more important to explore what's possible than to get a best answer.
2. The area of the sheet is 2500 square millimetres. Each of the 14 circles can use up to 178.57 square millimetres of silver, and so the maximum radius of each circle is 7.5 millimetres.

## Teachers' Guide to Worksheets

## Surface Area and Volume

As a whole class starter it may help to get the students to recall the formulas which will be used in the worksheets, and check that they understand what these mean and how to use them. The other teaching point in the worksheets is converting between different units: for example between cm and m .

These worksheets could also be used as part of a longer project with the worksheets on Business Equations.

## Surface Area and Volume Starter Worksheet

1. $\quad 113$ square centimetres
2. 7 centimetres
3. 75400 square centimetres
4. $\quad 12.6$ square metres
5. $\quad 0.785$ cubic metres
6. Total surface area $=2 \pi r^{2}+2 \pi r h=2 \pi r(r+h)$
7. 2.07 square metres

## Surface Area and Volume Core Worksheet

This worksheet is best used in groups, where there's a chance that other students will point out any mistakes such as failing to convert from cm to m or not getting the formula or calculation for the total area correct.

|  | Basic Mix | Superior Mix | Top Quality Mix |
| :---: | :---: | :---: | :---: |
| Cost of making <br> the tin | 73.5 p | 73.5 p | 73.5 p |
| Cost of filling the <br> tin with mix | 15.8 p | 19.8 p | 22.2 p |
| Total cost | 89.3 p | 93.3 p | 95.7 p |

## Surface Area and Volume Advanced Worksheet

The only possible height and radius combination that works is:
Height: 6 cm Radius: 5 cm (with volume 471 cubic cm , ratio 1.67)
Students then have to re-calculate the costs in the table above for this new height and radius. The total costs are:

Basic Mix: 61.3p
Superior Mix: 63.6p
Top Quality Mix: 65.0p

## Teachers' Guide to Worksheets

## Speed and Distance Graphs

The 15-minute film Pushing the Limits on this DVD is a story of how a British engineering company took the engines from a JCB road digger and got them to go as far as they possibly could. This DVD also includes a Flash animation car.swf which allows students to select speeds and acceleration values and experiment to see what happens to the graphs if they try different values. The worksheets are intended to support the animation.

There are several engineering companies who run challenges involving cars, including model cars run on sustainable resources such as solar power. The STEM directories (in maths, science, engineering) have further details on many of these.

## Speed and Distance Graphs Starter Worksheet

1. 

(a) $0 \mathrm{~m} / \mathrm{s}$
(b) $4 \mathrm{~m} / \mathrm{s}$
(c) $2.5 \mathrm{~m} / \mathrm{s}$
(d) $8 \mathrm{~m} / \mathrm{s}$
(e) $0.5 \mathrm{~m} / \mathrm{s}$
2. The speed in $\mathrm{m} / \mathrm{s}$ is the same as the time elapsed in seconds.

## Speed and Distance Graphs Core Worksheet

1. The car actually travels 18 metres.
2. The distance after each second is, respectively: 0.5 metres; 2 metres; 4.5 metres; 8 metres; 12.5 metres; 18 metres.

You may want to set your own targets for the investigation or simply allow students to experiment and then present their results to the whole class at the end. These resources might work best as part of a larger practical project working together with the science or design/technology department.

## Teachers' Guide to Worksheets

## Probability and Law

It is difficult to find good maths resources that link probability or statistics with law, and the worksheets are included as an example of what might be done with a class for whom English comprehension is not a barrier. In legal cases much of the evidence given to a jury is based around diagrams and graphs rather than in words, but advanced students may enjoy the chance to explore the underlying probabilities involved in the scenarios given.

If you have good ideas for maths resources set in a legal context, post them on the website of the National Centre for Excellence in the Teaching of Mathematics (www.ncetm.org.uk) on their communities forums. You can apply to them for small grants of money to help with projects designed to enhance mathematics teaching.

## Probability and Law Starter Worksheet

1. $5 \%$ (reading comprehension only)
2. 750 people ( $10000 \times 0.5 \times 0.15$ )
3. $\quad 0.075(0.5 \times 0.15)$
4. 0.02 ( 200 divided by 10000)
5. $15(0.02 \times 0.075 \times 10000)$
6. It's probably not reasonable to make the assumption - for example, some of the 10000 people in the town will be young children who wouldn't be out on their own. However it's likely to be at least 15 people, so the assumption can give us a lower bound.
7. Yes - at the moment there's no reason why it should be him rather than potentially at least 14 other people satisfying the evidence given.

## Probability and Law Core Worksheet

1. 30000 people (six times as many as in the database)
2. 0.0005 ( 30000 divided by 60000000 )
3. 0.000000667 ( 40 divided by 60000000 )
4. $3.33 \times 10^{-10}(0.0005 \times 0.000000667)$
5. It might be reasonable to make this assumption, but in a town people are more likely to be related to each other than when chosen randomly from the population. The statistical expert might be reasonably asked to provide a model of the probability to take this into account.
6. The estimated number of people who satisfy both conditions - assuming independence of the conditions - is 0.02 . Therefore finding someone who does satisfy both conditions is rather unlikely and indeed suspicious.
7. It would be helpful to know if Mr Robb and Mrs Steel have any other relations who were in the area at the time. It might be that it was in fact Mr Robb's identical twin brother (who would have the same DNA)!

## Teachers' Guide to Worksheets

## Averages / Presenting Data

Like the Calculating the Weather worksheets earlier, these worksheets focus on a meteorological context. The 15-minute film Weather Forecasting on this DVD features several weather forecasters explaining how they use maths in their work. Towards the beginning of the film you can see Jim Bacon draw a line representing an isotherm on a map of the UK.

It may be helpful to work with the science or geography department and see when and in what contexts they teach some of these ideas.

## Averages Starter Worksheet

The goal here is to get students to think about whether British summers are getting warmer. Global climate change may not mean that our summers in Britain do get warmer - they might instead get colder if the temperature rise changed the direction of the Gulf stream. This is a complex issue. However if students understand moving averages then it can help them understand the difference between natural variation and a general trend, and this is useful for any debates on climate change.

The moving averages are:14.7, 15.1, 14.6, 15.3, 15.0, 15.3, 15.9, 15.8, 15.8.

## Averages Core Worksheet

This worksheet could be used for revising averages.

1. The two missing temperatures are 12 and 15.
2. Map 1 has mean 13, mode 15 and median 13. (Friday) Map 2 has mean 15, mode 16 and median 15.5. (Thursday) Map 3 has mean 14, mode 12 and median 14. (Saturday)
3. 24 degrees.

## Presenting Data Starter / Core / Advanced Worksheet

## 2. 1120 metres (top left of map)

Most of these questions have graphical answers. It may be a good idea to get students to try to draw their answer on the board and discuss why they've done it that way. The advanced worksheet then tests whether they can carry out what they've learnt on a large map. It's important to make sure that the students understand that isotherms can't cross each other before they attempt this exercise.

