Keeping it cool: controlling the climate in railway stations

With a billion journeys taken on UK railways every year, railway stations are a vital part of our national infrastructure. Mathematics is helping to design the next generation of railway stations, ensuring they provide a comfortable environment for the travelling customer.

Work is under way on the largest engineering project in Europe: Crossrail. The project will connect 37 railway stations spanning the entire width of London. With the first sections planned to open in 2018, it is estimated that the service will ferry 200 million passengers across the capital every year. Current estimates place the economic benefit of Crossrail at £42 billion.

One of the largest of the new stations will be at Canary Wharf. Designed and built by Canary Wharf Group, the Station will be a large hub for London's city workers and residents. Above the Station will be 100,000 square feet of retail space and a new park covered by a spectacular timber lattice-roof. In designing such a busy station, retail and leisure space, it is important to ensure the passengers are comfortable. The way the station is constructed has an effect on whether the station is stagnant, windy, hot or cold. It is key, then, to ensure the design of the station is right before starting on its multi-million pound construction. This is where mathematics can help.

As with the existing Tube station at Canary Wharf, the Crossrail station is being built in a dock and a balcony will overlook the water. Underneath the balcony are large extractor fans that are necessary to remove the heat generated by the trains in the tunnels below. The air flowing from these fans can reach 60°C. It is therefore important to make sure that this hot air is not flowing back into the station where it would raise the temperature within. Engineering and design firm Arup have used mathematics to check and modify the design of the station to help direct this flow of hot air.

First the engineers need to determine the temperatures of the surfaces (walls, floors and ceilings) of the station – known as boundary conditions. These boundary conditions are dependent on factors such as the original temperature of the air outside and inside the station, the heat from people, the weather and trains. Equations governing these temperatures can be run through a computer to see what the conditions will be after a small amount of time (called a time-step). This process is repeated to model the conditions over any given time. A dynamic thermal model is used that is solved numerically using what is known as a forward finite difference method.

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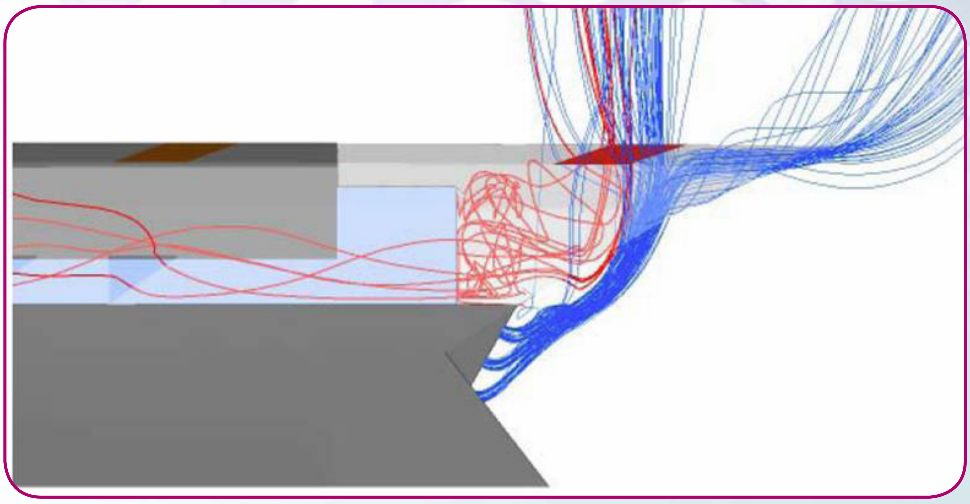
In the dynamic thermal model, the heat conduction occurring in the station walls, floor and ceiling is visualised as a series of resistors and capacitors – elements that can resist and store heat respectively. Large concrete sections are prime examples of capacitors because they can store the heat, before releasing it later in the day. The model is run over consecutive ten minute intervals to see how changes in overall temperature in the station and those of the station surfaces unfold over time. Several iterations are run as the boundary conditions are dependent on the time of year – they will be different between winter and summer, for example.

Once the temperatures of the surfaces are known, a detailed model is needed to show how the hot air entering the station interacts with the station surfaces and environment. For example, whether the heat is dissipated quickly having little overall effect, or whether it stays around potentially causing an unacceptable rise in overall temperature. The flow of fluids in this way is described by a set of equations called the Navier-Stokes equations, named jointly after the mathematicians who came up with them independently in early 19th century.

There is, however, a catch: fluids are made up of countless particles all moving together in three dimensions. This complexity renders the equations almost impossible to solve directly. In fact, the problem forms one of the Clay Mathematics Institute's Millennium Maths Problems, with $1 million offered for a solution. Faced with such a difficult task, engineers, mathematicians and physicists have to break the problem down into chunks that can be churned through large computer clusters to give an approximate solution. This type of model is a form of computational fluid dynamics.

If it turns out that the hot air from the ventilation fans is providing an unacceptably high contribution to the station temperature then the design can be adjusted to reduce the flow into the station. The same mathematical techniques are used by Arup and other companies in the effective design of offices, sports stadiums, schools and hospitals. In this way mathematics is not only helping to keep commuters comfortable on their journeys but plays a part in the infrastructure we all encounter on a daily basis.



TECHNICAL SUPPLEMENT

Forward Finite Difference Method

Differential equations are difficult to solve analytically. Finite difference methods are used to approximate the first order derivative of a function, meaning differential equations can be represented by a set of linear equations that are approximately the same. These equations are then much easier to solve. The approximation is based on sampling the values of the function around the desired point. In forward finite difference methods the sample value has a later value than the point for which the equation is being solved.

Navier-Stokes Equations

The Navier-Stokes equations were first derived by the French physicist Claude-Louis Navier in 1822, but later developed independently by the British mathematician George Stokes in 1845, who wrote the equations in the form still used today. The equations are derived from Newton's second law of motion (force equals mass times acceleration) and describe the relationship between the velocity, pressure, viscosity and density of a moving fluid. As a linked set of four non-linear partial differential equations, the Navier-Stokes equations are impossible to solve analytically in almost all cases, hence the need for numerical approximation methods.

References

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