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THE BINOMIAL THEOREM?

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The binomial theorem and taste testing

When you learn algebra at school, it's not long before your teacher starts making you 'expand brackets'. You then never escape from these expanding brackets and one day you may find yourself learning about the binomial theorem. But why would anyone need to know about the binomial theorem?

The binomial theorem is a short cut so that you don't have to expand brackets and then simplify the terms. Mathematicians from Euclid in 400 BC onwards have noticed this short cut. It was eventually formalised by Blaise Pascal in a pamphlet that was published in 1665, shortly after he died.

If you are surveying people to find out which cola drink they prefer you could get lots of people to choose between the two products and see which one gets picked the most. If cola A is selected more than cola B, you could conclude that cola A tastes better. But what if they actually taste equally good? It could have been that cola A was selected more often just by random chance.

If the two drinks actually taste equally good, then they each have a 50% chance of being selected. So what if four, or even five, out of five people selected cola A?

Is this enough to mean that it's better or could this be random chance?

If you let a stand for the probability that someone prefers cola A and b for the probability that someone prefers cola B, then you can represent five people making a choice like this:

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

For the situation where cola A and cola B have the same probability of being chosen, $(a = b = 0.5)$ you get the following values:

Number of people choosing cola A	Number of people choosing cola B	Probability
5	0	0.031 25
4	1	0.156 25
3	2	0.312 50
2	3	0.312 50
1	4	0.156 25
0	5	0.031 25

So this means that the probability of four or five out of five people choosing cola A is 0.1875 or 18.75% when the colas taste equally good. You should be very suspicious of the conclusion that cola A definitely tastes better.

Relativity

Part of the power of the binomial theorem is its ability to speed up the use of complicated equations. In the equations that govern Einstein's theory of relativity there is this term.

$$\frac{1}{\sqrt{1-x}}$$

We can use the binomial theorem to expand this.

$$\frac{1}{\sqrt{1-x}} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \ldots$$

When a satnav uses GPS coordinates to calculate its position, the GPS satellites are moving in a different gravitational field to your car and so the calculations need to allow for relativistic time dilation, where time appears to move at different speeds when observed from different moving objects. Thanks to the binomial theorem, a satnav can do these calculations quickly enough to get you home on time!

Making pictures

If you write the coefficients of the binomial theorem in a triangle then you end up with Pascal's triangle, where each number equals the sum of the two above.

Then, if you colour in all of the odd numbers, you end up with the fractal known as the Sierpinski triangle. The Sierpinski triangle is created by taking a triangle, splitting it into four equal triangles and removing the middle one, then continually repeating this process on the new triangles created.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1