

what's the point of...

GEOMETRY?

Fighting back against cancer



One of the most deadly yet least understood diseases known to mankind is cancer. Cancer causes cells in the body to grow uncontrollably and the malignant growth may invade other tissues and organs in the body.

Geometry is a branch of mathematics that can help identify cancerous growth and help in the prevention and cure of the disease.

Although cancer cells are three-dimensional, by studying cell structure in two dimensions we can determine whether or not further investigation is required. For example, mammograms are used to screen for abnormalities or to identify the nature of breast lumps.

Cancer cells derive their name from the Greek word for 'crab' because they are often crab- or star-like in appearance. The breast is x-rayed and the cell clusters shown on the resulting mammograms are analysed. If irregular or star-shaped clusters are found, the ratio of

$$\frac{p^2}{a}$$

where p is the perimeter of the cluster and a is the area of the shape, can be determined. The larger the ratio, the greater the concern.

Extending the technique into three dimensions allows comparison of the surface area to volume ratio and offers a different perspective and, possibly, ideas for dealing with cancer.

Investigation

You can investigate the ratio of the perimeter of a shape to its area by taking various measurements of regular shapes (such as squares, pentagons and hexagons) and calculating the ratio

$$\frac{p^2}{a}$$

Extend this to irregular shapes. What do you find?

With regular shapes, you may notice that as the number of sides increases, the ratio decreases. However, with irregular shapes, it is not so easy to generalise and in some cases you will see the ratio increase, meaning further examination of the cell clusters in real life.

What's that coming over the hill? Is it a locus?

In mathematical terms, a locus is a collection of points that share a common feature or property. For example, a circle is made up of a locus of points that are equidistant from a fixed point (the centre).



This definition can be extended further. For example, the locus of zero values given by a quadratic polynomial (not tackled here!) gives rise to some spectacular shapes called quadratic surfaces with names such as spheres and saddles.

If we delve deeper still we enter the realms of fantasy and can explore impressive geometrical representations such as fractals. Elsewhere in this booklet you can see the Sierpinski triangle and the Mandelbrot set. You can also see fractal geometry in everything from crowd movement at a major sporting event to snowflakes to computer-generated imagery for games and movies (such as is seen in the opening sequence of *Casino Royale*).

There is a simple fractal that you can create that will also give you an insight into the mysterious world of infinity. One mathematician who explored this was Georg Cantor (1845–1918). He produced the Cantor set using the following method.

The Cantor set

1. Start with a line of length 1. This can be defined as the set of points $0 \leq x \leq 1$.
2. Underneath this draw a copy of the line with the middle third removed
(i.e. $\frac{1}{3} < x < \frac{2}{3}$ removed.)
3. Underneath this draw a copy of the last line with the middle third of each section removed.
4. Repeat stage 3 (infinitely many times!).

Two questions to think about.

- What is the sum of all the 'lengths' of the regions you have removed?
- Can you identify any region or point from the original line 'length' that has not been removed?

The sum of all the lengths removed is the infinite series

$$\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \dots,$$

which totals 1. In other words, you started with a line length of 1 and, if you sum the lengths of all the removed regions, that should also come to 1. You may well be thinking that you must have no points left but, and here's the brain-frying bit, take the point

$$x = \frac{1}{4} :$$

this point can never be removed using the process above.

In fact there are an infinite set of points that aren't removed by this process. These are the Cantor set. The Cantor set, by logic, should be empty but, by logic, it should also contain an infinite number of points!