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Knowledge Transfer Network
Industrial Mathematics

Taking a new turn

HYBRID APPROACHES FOR REAL-WORLD CONSTRAINED PORTFOLIO SELECTION PROBLEMS

Numerical Algorithms Group / University of Nottingham

The need

One of the most studied subjects in financial mathematics is the portfolio selection problem. In its most basic formulation, the aim is to choose a set of assets (or equities) from a basket of those available, in order to maximize the expected rate of return, given the acceptable amount of risk. Naturally, equities that offer the prospect of high returns are also likely to be associated with high risk.

Current models ignore many aspects of real-world trading. However, when extra constraints are built in to allow this, it becomes more difficult or impossible to solve the problems using standard library software. The aim was then to design and implement a hybrid solution scheme combining heuristic and exact methods.

The outcomes

The basic portfolio selection model ignores many aspects of real-world trading. For example, it assumes that

- there are no transaction costs or taxes (costs associated with buying or selling a stock)
- there are no minimum or maximum stock order sizes
- the correlations between assets are constant (in reality they are not)
- there are no ceiling constraints designed to avoid excessive exposure to a specific asset

The intern included some of these constraints and solved the new problem using a hybrid solution scheme.

After the completion of the internship, the intern plans to investigate the features of a more complex and large-scale portfolio rebalancing problem supplied by the Numerical Algorithms Group (NAG). The model involves a more complex risk measure using non systemic risk and liquidity risk. More efficient modelling and solution approaches are still to be determined.

"Fang's valuable work demonstrated the practical benefits of hybrid methods and will lead to enhancement of our library software in future. Importantly, her direct involvement with our customers helped improve NAG client relationships."

Mick Pont
Numerical Algorithms Group

Technical summary

It is normal to solve the portfolio optimisation problem as an integer programming problem because it is not usually possible to buy fractional amounts of stocks. Also note that a negative value for any of the buy variables signifies a short position, i.e. the selling of stocks that are not yet owned.

A well-known financial institution provided a problem for study. The problem was solved by a NAG general nonlinear programming subroutine. Further technical support was provided to the users, when they decided to implement the solution method themselves.

After understanding the requests of several other customers, the extension of the classical Markowitz mean-variance portfolio selection problem (PSP) was investigated. The real-world trading constraints were considered simultaneously in a single model. These trading constraints include a cardinality constraint that limits the total number of assets which can be held, a buy-in constraint that avoids holding small positions, and the constraint of buying assets by lots.

These constraints were modelled with integer variables (both binary and general integer variables) that led to a Mixed Integer Quadratic Program (MIQP). To solve the resulting MIQP problem efficiently, a Layered Branch-and-Bound (Layered B&B) algorithm was proposed, in which the B&B tree was layered according to different features of the variables.

The Layered B&B is a kind of decomposition where the search is performed on the top layer first before it goes down to further layers. Two branching heuristics and one node selection heuristic were proposed to individual layers of the B&B tree to get the optimal solution faster. The Layered B&B algorithm was implemented and tested on a benchmark problem with real-world trading constraints. The results show the efficiency of the algorithm.

"It was a very good experience for me to work at NAG. I have gained valuable knowledge of real-world problem modelling and solving. It was great to apply OR techniques and advanced NAG routines to solve the real-world problems."

Fang He
University of Nottingham

"It's been highly valuable to work on real-world problems, and see research outcomes being applied towards bridging the gaps between theory and practice."

Rong Qu
University of Nottingham

This project was part of the programme of industrial mathematics internships managed by the Knowledge Transfer Network (KTN) for Industrial Mathematics. The KTN works to exploit mathematics as an engine for innovation. It is supported by the Technology Strategy Board, in its role as the UK's national innovation agency, and the Engineering and Physical Sciences Research Council, in its role as the main UK government agency for funding research and training in engineering and the physical sciences.

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Engineering and Physical Sciences Research Council

Project Details

Partners
Numerical Algorithms Group
University of Nottingham

Project investment
£12,000

Intern
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