Technology Strategy Board
Driving Innovation

Knowledge Transfer Network
Industrial Mathematics

A new lease of life

ESTIMATION OF DROPLET COLLISION RATES IN A SPRAY DRYING TOWER

Procter & Gamble / University of Strathclyde

The need

Detergent spray drying towers bring atomised slurries into contact with hot air to form powdered laundry detergent particles. As well as drying in the tower, the droplets also collide and agglomerate as they are transported through the tower. The extent of the agglomeration can limit the capacity of the towers. P&G wanted to produce a mathematical model of the collision and agglomeration and other phenomena to help predict particle sizes that are produced.

Previously, computational fluid dynamics (CFD) approaches have been used to look at the problem of tower agglomeration which require the movement of each particle in the spray to be tracked individually. This project looked to develop a new approach to the problem.

The outcomes

Understanding how particles collide and agglomerate in a detergent spray drying tower is very important to P&G. Being able to determine which operating conditions affect the agglomeration will allow P&G to decide which operating conditions will produce the 'best' size range of particles coming out of the tower.

A mathematical population balance model has been developed to describe the agglomeration in the spray zone of the tower. Analytical and numerical solutions of this model have been investigated. There is scope to develop the numerical model further to incorporate other processes occurring in the tower.

A numerical technique has been implemented to solve a population balance model for the agglomeration process in the spray zone. Quantitative physical data can be input and the output can be used to help decide which operating conditions affect the agglomeration process. It is possible that the model can be adapted further to include other processes such as breakage and drying.

The PhD intern has increased her knowledge of the techniques used to solve population balance equations which are the basis of her PhD project.

"This was our first collaboration with a Mathematics department. It has given us a fresh perspective on the problem, which complemented our engineering view, and has both helped us understand the problem and its potential solutions."

Andrew Bayly
Procter & Gamble

Technical summary

It is thought that most of the agglomeration occurs in the spray zone of a detergent spray drying tower so the model concentrated on the processes occurring in this area. It was decided that the intern should work with a one (spatial) dimensional model – the spatial dimension being the distance down the spray. It was assumed that there is no breakage of the particles and that they could only agglomerate. It was also assumed that all particles enter the spray with the same initial speed and that the initial particle distribution is well mixed. The effects of drying were not taken into account.

A population balance equation (PBE) was used to model the change in number concentration of particles of different sizes due to the process of advection and agglomeration. This equation took the form

$$\frac{\partial n(x;r,t)}{\partial t}+\frac{\partial[U(x;r)n(x;r,t)]}{\partial r}=$$
$$-2\frac{U(x;r)n(x;r,t)}{r}+\frac{1}{2}\int_0^x K(x-y,y;r)n(x-y;r,t)n(x;r,t)dy$$

where n is the number concentration of particles of size x at a position r down the spray at time t, U(x;r) is the speed of a particle of size x dependent on its position down the spray and K(x,y;r) is the collision rate for particles of size x and size y at position r.

Analytical solutions exist in the literature for the pure coagulation equation (U=0) for certain kernels, e.g. K(x,y)=1 or K(x,y)=x+y. In the case of the steady-state version of the PBE (when the time derivative is set to 0) similarity solutions for more complicated kernels were investigated. These similarity solutions give information on the asymptotic behaviour (large r) of solutions.

Numerical approaches were then investigated. The fixed pivot and cell average techniques, which have been developed to solve population balance models, were implemented and were tested against the analytical solutions. As seen in the literature, the cell average technique proved the most accurate. This technique was coupled with a second order upwind discretisation of the advection term to obtain a numerical scheme for dealing with the full PBE. The physical equations for particle speed and collision kernels were then researched and implemented into the code. This resulted in a numerical method for solving the whole equation with different physical input parameters.

"This project has given me an insight into what it would be like to work as part of a global company. I have also learned a number of mathematical techniques which were previously unknown to me."

Louise Smith
University of Strathclyde

"Through the KTN internship with P&G, we were able to carry out analytical and numerical investigations into a population balance model of particle agglomeration in a spray drying tower. The opportunity to work with P&G has given us valuable insight into the type of coagulation-fragmentation problems that arise in an industrial context."

Wilson Lamb & John MacKenzie
University of Strathclyde

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EPSRC
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Project Details

Partners
Procter & Gamble
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£15,000

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