



TRIGONOMETRY

INTEGRATION

LOGARITHMS

PROBABILITY

what's the point of... **QUADRATIC EQUATIONS**

SEQUENCES

TRIGONOMETRY

INTEGRATION

LOGARITHMS

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QUADRATIC EQUATIONS

**Ever wondered what's the point of maths? Did you know that quadratic equations and probability can be linked to football and that trigonometry can be linked to combating terrorism? In fact, maths is used in a huge number of practical ways – more than you may initially think.**

This booklet has been designed to highlight the ways in which maths is used practically in daily life in professional and leisure scenarios. The case studies guide you through the practical ways in which maths can be used from question to discussion and highlight the many ways in which it can be applied to an everyday setting.

This booklet has been produced by *more maths grads*. *more maths grads (MMG)* is a three-year project funded by the Higher Education Funding Council for England to develop, trial and evaluate means of increasing the number of students studying mathematics and encouraging participation from groups of learners who have not traditionally been well represented in higher education.

MMG has also been working with the Higher Education Funding Council for Wales to translate and disseminate its resources throughout Wales. This booklet is just one example of the translated resources.

# what's the point of... INTEGRATION?

## As easy as pi

**You're a Greek philosopher in the year 225 BC. What's the area of a circle with given radius?**

**You're a wine merchant in Austria in the year 1615. Which shape of barrels will hold the most wine?**

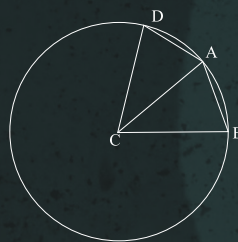
**You're designing a new type of airbag to prevent head injury in car crashes in 1955. Does it work?**

**You're a particle physicist in 1989. How much force do you need to separate two electrons?**

**You need to create a better version of JPEG compression for image files. What maths will be useful?**

Integration helps us to answer each of these questions. Integration is closely associated with its opposite process, differentiation. Together they are known as calculus. Related ideas have been studied for at least two thousand years. The idea of integration is based on calculating an area or volume by adding up lots of small areas or volumes that are easier to compute.

Suppose you have a circle with radius  $r$  and you've forgotten that the formula for its area is  $A = \pi r^2$ . You could work out the area roughly by filling the circle with triangles and calculating the area of each triangle. This is what Archimedes did over two thousand years ago to work out a better estimate for the value of  $\pi$ .



## Give me a place to stand and I will move the earth

One of the greatest mathematicians of all time, Archimedes was born in Sicily in the Mediterranean in 287 BC and was killed in the Roman invasion in 212 BC. In between he figured out a huge amount about mathematics and physics, and designed a water pump that is still in use in Egypt today.

He once said to his friend King Hiero, "Give me a place to stand and I will move the earth." The king challenged him on this. Archimedes then chose a ship which needed many men to move it out of the dock, set up a pulley, and was able to move it himself without much effort.



Archimedes also showed that the exact value of  $\pi$  lies between the values  $3^{10/71}$  and  $3^{1/7}$  by drawing two regular polygons with 96 sides,

one inside a circle with its corners on the circle (inscribed) and one outside the circle with its sides just touching the circle (circumscribed). Modern integration was born out of ideas like this.

## Eighteen hundred years later...

Johannes Kepler lived in central Europe. He worked on data gathered by the Danish astronomer Tycho Brahe and figured out that the planets moved in elliptical – not circular – orbits around the sun. This is why sometimes Pluto is closer to the sun than Neptune – its orbit is more squashed.

He noticed that planets travel faster at some points on the orbit. The line joining a planet to the sun sweeps out the same area in a given interval of time, no matter where the planet is. This means that the planet must move faster when it is closer to the Sun.



At his second wedding, Kepler got distracted trying to figure out a better way to work out the volume of the wine barrels there. He wrote a book on the subject in 1615.

In both these problems Kepler used the idea of splitting up an area or volume into smaller parts in order to compute it. This is the key idea of integration.

For further information, articles and resources visit:  
www.moremathsgrads.org.uk • www.mathscareers.org.uk  
plus.maths.org • nrich.maths.org • www.cs4fn.org

Written by Zia Rahman and Vivien Eason, More Maths Grads, School of Mathematical Sciences, Queen Mary, University of London (QMUL)  
Special thanks to Professor Peter McOwan (QMUL), Professor David Arrowsmith (QMUL), Makhani Singh, Melanie Ashfield and James Anthony, University of Birmingham

# Preventing injury in crashes

**You're travelling in a car along a city street at 30 mph. What happens if you have to brake suddenly?**



Usually it takes 1.5 to 2 seconds to stop a car when braking normally. However in a violent impact, such as a car crash, it can take as little as 0.1 seconds to stop a car. This can cause serious head injuries.

Since the 1950s, many cars have come equipped with airbags in the dashboard. These help prevent head injuries by slowing down the deceleration of the people in the car.

In tests of airbags, a calculation is made called the Head Injury Criterion, or HIC for short. If the test gives a HIC value above 1000 then the crash would have been life-threatening. Modern cars may have HIC values of 100 to 200. The HIC is calculated by looking at every possible time interval between

start time,  $t$ , and stop time,  $s$ , during the braking period and finding the average deceleration for each of those time intervals. To find the HIC we take this average deceleration raised to the power 2.5 (based on car crash data) and multiply it by the length  $(s - t)$  of the time interval. The HIC is the maximum over all possible time intervals  $[t, s]$ .

How do you find the average deceleration? It is the integral of the deceleration, divided by the length of the time interval.

The deceleration at time  $t$  can always be found, either by integrating or by approximating the area under the curve at that point.

Now imagine the maths that Formula One engineers use to make sure their cars stay on the road even when travelling at 200 mph!

## Keep it down!

**There are many more applications of integration and of calculus. The JPEG 2000 image compression standard is based on wavelet theory which uses a lot of integration. Image compression ensures your photo files take less memory per image.**

Calculus is needed in physics to calculate the effects of forces on tiny particles or in massive galaxies. Economists use integration techniques to model stock prices.

Integration equips you with the essential skills necessary for either a technical or scientific profession!

## Websites to check out:

[www.mathscareers.org.uk](http://www.mathscareers.org.uk)

[plus.maths.org](http://plus.maths.org)

*Interview with maths student:*

*"If I've got a maths degree, I can be pretty much anything!"*

<http://plus.maths.org/issue28/interview/index.html>

The MacTutor History of Mathematics Archive at the University of St Andrews:

[turnbull.mcs.st-and.ac.uk/history/](http://turnbull.mcs.st-and.ac.uk/history/)

what's the point of...

# INTEGRATION?

what's the point of...

# LOGARITHMS?

## Disaster prevention: understanding earthquakes

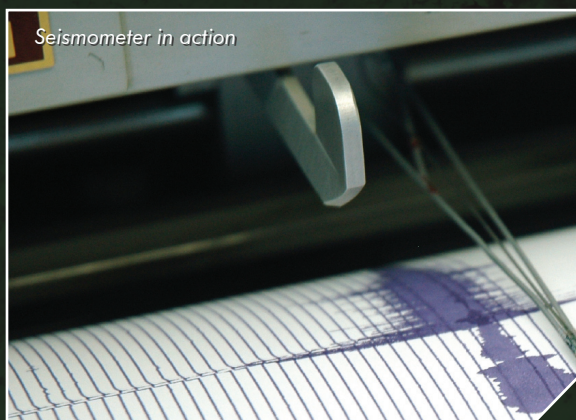
**On the 8th October 2005, a major earthquake struck a mountainous region of South Asia. The shock waves radiated out from the epicentre of the earthquake, about fifty miles north-east of Islamabad, the capital of Pakistan.**

It wiped out many villages and left over three million people homeless. Over seventy thousand people died in Pakistan and in the Indian-administered state of Jammu and Kashmir.

On the 23rd September 2002, a minor earthquake hit the United Kingdom. The epicentre was in Dudley in the West Midlands, north-west of Birmingham. A few homes were damaged but no-one was injured.

### How much stronger was the first earthquake than the second?

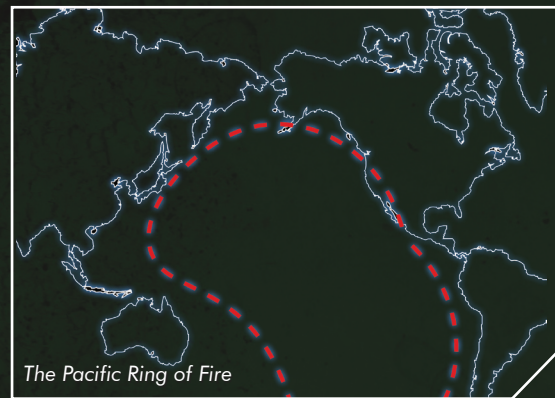
You can measure the strength of an earthquake by using a seismometer. The seismometer measures how much the earth shakes and records it as a graph. Stronger earthquakes have graphs which go up and down more: you can say that the maximum difference in height, which is called the amplitude of the graph, is bigger. This amplitude tells you how strong the earthquake is.



### Where do earthquakes happen?

Nine out of ten earthquakes happen along the Pacific Ring of Fire, which circles the Pacific Ocean. Japan, California, Chile and the Philippines all lie along this ring. Seventy years ago two earthquake scientists, Charles Richter and Beno Gutenberg,

were working in California. They wanted a way to tell how many of the earthquakes in California would be big ones causing serious damage. They decided to give each earthquake a magnitude number. An earthquake with a higher number would be more serious than one with a lower number. The earthquakes mentioned earlier were measured at 7.5 (South Asia) and 4.8 (UK).



### How do you calculate the magnitude of an earthquake?

These numbers are calculated by taking the amplitude of the largest wave, taking its logarithm to base 10, and then adding a factor which depends on the distance between you and where the earthquake is. Because the scale is created by taking logarithms to base 10, an earthquake with magnitude number 7 will be ten times stronger than a magnitude 6 earthquake.

### How much stronger was the Asian earthquake?

We take the difference between their magnitude numbers and get  $7.5 - 4.8 = 2.7$ . Therefore 2.7 is the logarithm to base 10 of the number we want. If we calculate 10 to the power 2.7 on a calculator we get 501.19. Try it out for yourself. This means that the Asian earthquake was five hundred times stronger than the one in the West Midlands.

### Why do people use logarithms here?

It's much easier to talk about earthquakes with magnitude 6.5 or 9.0 than to talk about earthquakes with 5 000 000 or 32 000 000 000 tons of energy.

For further information, articles and resources visit:  
www.moremathsgrads.org.uk • www.mathscareers.org.uk  
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# Apple juice, coffee, milk and soap

**Another scale which uses logarithms is the pH scale which measures how acidic a liquid solution is. An acid such as vinegar has a pH value of around 3.**

The opposite of an acid is an alkali. Alkalis include soap and bleach. Chemically, an alkali cancels out an acid. Since many stains on clothes are acidic – tea, coffee, apple juice, milk – washing powders and bleaches are usually alkaline. Household bleach has a pH value of around 12.5.

Somewhere in between 3 and 12 on the pH scale we find solutions with a pH of 7. The pH of pure water is 7. Anything with a pH of less than 7 is called an acid; anything with a pH of more than 7 is called an alkali.

Just as for measuring earthquakes, this scale is logarithmic. This means that an acid such as lemon juice with a pH of around

2.5 is ten times more acidic than an acid such as orange juice with pH 3.5. Even your skin is slightly acidic. The soap in your bathroom probably has a pH value of between 9 and 10 so it'll help remove the sticky orange juice but won't react much with your skin. The bleach would be about a thousand times stronger, which is why you don't put it directly on your hands!

Once again, using logarithms helps us use a scale of numbers which is faster to write down.

## Experiment

Get a can of cola and some dirty 1p and 2p coins. Leave the coins in a glass of cola overnight. Next morning take your coins out of the glass. The acid in the cola will make your coins look new and shiny! Why? Cola contains phosphoric acid – it's as acidic as lemon juice!

## Interesting times

**How much does your favourite snack cost? It probably costs a bit more than it did a few years ago. This is due to inflation – in a healthy economy prices creep up slowly. To make up for this, employers usually give their employees a cost-of-living increase in their wages each year.**

What about people who save money? Banks will pay interest on your savings so that they also increase in value. They might pay it monthly, or every three months, or once a year. Which is best?

Suppose that you have £5000 in the account and the bank pays 5% annual interest, and computes it every six months. After six months you would have  $£5000 \times \sqrt{1.05} = £5123.48$ . After a year you would have  $£5123.48 \times \sqrt{1.05} = £5250$ .

What if banks calculated interest differently, finding the interest paid every six months by halving the annual interest rate? How much would you have after three years?

$£5000.00 \times (1 + 0.05 \times \frac{1}{2}) = £5125.00$  after six months.  
 $£5125.00 \times (1 + 0.05 \times \frac{1}{2}) = £5253.13$  after one year.  
 $£5253.13 \times (1 + 0.05 \times \frac{1}{2}) = £5384.46$  after 18 months.  
 $£5384.46 \times (1 + 0.05 \times \frac{1}{2}) = £5519.06$  after two years.  
 $£5519.06 \times (1 + 0.05 \times \frac{1}{2}) = £5657.04$  after 30 months.  
 $£5657.04 \times (1 + 0.05 \times \frac{1}{2}) = £5798.47$  after three years.

0.05 corresponds with the 5% rate. We also multiply by  $\frac{1}{2}$  because six months is half of a year. The interest is rounded to

### Websites to check out:

[www.mathscareers.org.uk](http://www.mathscareers.org.uk)  
[plus.maths.org](http://plus.maths.org)

Interview with a financial engineer:  
[www.plus.maths.org/issue46/interview/index.html](http://www.plus.maths.org/issue46/interview/index.html)

History of the number e and of logarithms:  
[www-history.mcs.st-andrews.ac.uk/HistTopics/e.html](http://www-history.mcs.st-andrews.ac.uk/HistTopics/e.html)

the nearest penny. The final amount is £5798.47. What would happen if the bank computed your interest every month, or every day?

Final amount after three years if interest is paid on £5000 or on £10 000.

INTEREST PAID EVERY:	INITIAL AMOUNT	
	£5000	£10 000
One year	£5788.13	£11 576.25
Six months	£5798.49	£11 596.93
Three months	£5803.84	£11 607.55
Each month	£5807.54	£11 614.72
Twice a month	£5808.66	£11 616.53
Every day	£5809.11	£11 618.22
Every hour	£5809.17	£11 618.34
Every minute	£5809.17	£11 618.34
Every second	£5809.17	£11 618.34

If interest is paid more frequently, you get more. However, after a point, the extra amount gets so small as to not make a difference. Computing the interest over increasingly smaller time intervals does not result in any extra money. The maximum value you can get is the original amount multiplied by 1.161 833 7. If you take the logarithm of this to the base e (where  $e = 2.718...$ ) you get 0.15, which is  $3 \times 0.05$  (number of years multiplied by the interest rate). This is true for any period and any interest rate. Logarithms are used a lot in investment banking for making financial calculations like this.

The number e, which equals 2.718281 8..., is special in mathematics. It was first discovered in 1683 by Jacob Bernoulli, a Swiss mathematician who wanted to understand the compound interest problem. But it is also special because the function  $y = e^x$  differentiates to itself, and for many other reasons.

what's the point of...

## LOGARITHMS?

what's the point of...

# PROBABILITY?

## Oh no...penalties...again!!!

**In the summer of 2008, football fans could follow Euro 2008 without the stress of seeing any of the home nations knocked out on penalties (because they never managed to qualify in the first place).**

Take England. Out of the last eight major tournaments that they have qualified for they have gone out on penalties five times (being knocked out by other means the other three times). This raises an interesting question – as the opposition manager about to play England, should you play for penalties?

In total, England have been involved in seven penalty shoot-outs in competition and have lost six of them – their only success coming against Spain in Euro '96. So is this 14% success rate statistically significant? How can England improve the odds of success in penalty competitions? Penalties are supposed to be a hit and miss affair – but with a bit of practice and some mathematical analysis, England may well overcome their penalty-taking curse.

Let's set up a simple scenario when taking a penalty.

- A striker can shoot either to his/her left or right, and similarly a goalkeeper can dive to his/her left or right.
- If the goalie dives to his/her left and the striker shoots to his/her left OR if the goalie dives right and the striker shoots right then a goal is scored (assuming the striker is accurate) because the goalie will be diving away from the ball.
- If the goalie dives to his/her left and the striker shoots to his/her right (or vice versa) then the goalie and the ball are reasonably close together and there is a 50% chance the goalie will save the ball.
- Let's assume that the striker is accurate when shooting left 70% of the time and 90% when shooting right.

Using mathematics we can estimate the best strategy for the striker to employ – it involves shooting to his/her left 56% of the time and to the right 44% of the time, irrespective of the goalkeeper's strategy. Overall this corresponds to scoring around 60% of the time. But why should the striker shoot more to his/her left side even though this is less accurate (70%) than when shooting to the right (90%)?

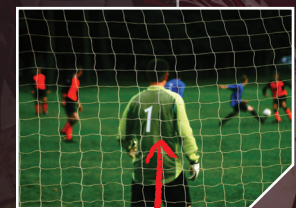
Using the same mathematics we can also estimate the best strategy for the goalkeeper – it suggests diving to

his/her left 69% and to the right 31% of the time. So if the striker shoots to the more accurate right side, the goalkeeper will dive more often to his/her left and increase the chances of saving the shot. However if the striker shoots to the less accurate left side, the goalie will only dive in this direction (to his/her right) around 30% of the time – so the lower shot accuracy is compensated for by the fact the shot is less likely to be saved because of the goalkeeper's strategy.

(For a more in-depth perspective on the maths, please see the article by John Haigh on *Plus* magazine website: <http://plus.maths.org/issue21/features/haigh/index.html>)

Of course, penalties are blasted into the back of the net or accurately placed. They may be in the top left corner, straight down the middle or in the bottom right corner. The goalkeeper may elect not to dive at all or may find that reaching a penalty to the top left is more difficult than reaching a penalty aimed to the bottom left. But at this stage you simply construct a more realistic model involving more than just shooting left and right.

So practice is the better alternative, but the maths and statistics can help analyse performances. In fact, think of all the stats that underline a good performance – not just penalty taking – the distance covered by Steven Gerrard in a match, the number of tackles by Cesc Fabregas, the pass accuracy of Lionel Messi or the power of a shot by Cristiano Ronaldo – it all counts ...



Which angle would you pick?

For further information, articles and resources visit:  
[www.moremathsgrads.org.uk](http://www.moremathsgrads.org.uk) • [www.mathscareers.org.uk](http://www.mathscareers.org.uk)  
[plus.maths.org](http://plus.maths.org) • [rich.maths.org](http://rich.maths.org) • [www.cs4fn.org](http://www.cs4fn.org)

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# The long arm of the law - probably

**In 1999, Sally Clark was tried, convicted and sentenced to life imprisonment for the double murder of her two sons who were aged just 11 weeks and 8 weeks at the time of their deaths.**

The tragedy shocked the nation, as the expert testimony of Professor Roy Meadow indicated that the chances of the double deaths happening in the same family from natural causes – Sudden Infant Death Syndrome (SIDS) commonly known as cot death – were 1 in 73 million. In other words, so unlikely that Sally Clark must be guilty of the murder of her sons.

However doubts surfaced about the testimony of the expert witness on the grounds of poor mathematical reasoning. The Clarks had always protested their innocence and there was much debate about the testimony; the Royal Statistical Society had issued a press release pointing out the mistake and indeed the conviction was quashed in 2003.

So what happened? If two events are considered to be unconnected they are said to be independent of each other. Professor Meadow made the (invalid) assumption that the two cot deaths were independent. For a non-smoking, affluent family the chance of a cot death occurring is around 1 in 8500. So to calculate the probability of two deaths occurring in one family he simply multiplied the probabilities together giving a result of 1 in 73 million. He then presented this as the probability that Sally Clark was innocent. This is a case of the Prosecutor's Fallacy. Are you guilty given the evidence or given the evidence are you guilty?

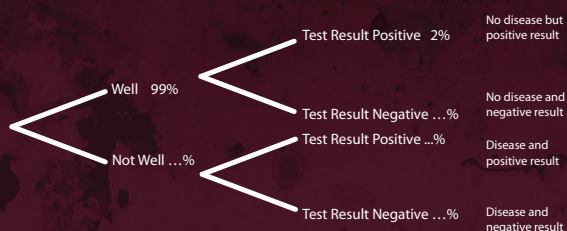
However, research suggests that in a family where one episode of cot death has occurred, the chance of it happening to another sibling is increased by between 10 to 22 times – this means that two cot deaths are certainly not independent. Also consider this, in normal circumstances the probability of either double SIDS or double murder in a single family is very small but, given that a double death has actually occurred, the chances of it being double SIDS or double murder are more likely.

## Are you well? Probably

**Medicines that come to the market have done so on the basis of rigorous testing and statisticians are vital to that role.**

Pre-clinical trials produce masses of data that must be carefully analysed to determine safety. Clinical trials involving people can take a number of years and include the design of safe trials, the right dosage of medicine and other factors.

Suppose we undertake a screening programme to identify a disease and hence administer a cure. The aims are quite reasonable. Now suppose 1% of the group suffer from the disease and the rest are well but also that there is a 2% chance that the test produces a false result. Using this information can you complete the following probability tree diagram?



By moving along the branches we can calculate the various probable outcomes and fill in the probabilities associated with each outcome. The two 'dodgy' outcomes are small enough to be considered

acceptable. The probability of being well but having a positive test result is known as a False Positive, and the probability of having the disease but having a negative test result is known as a False Negative.

However, in real life the medication we need to administer is potent and expensive. Consider everyone with a positive test result. How many of them actually have the disease? Using the probabilities given, we see that the probability of having a positive result is 2.96% whereas the probability of having a positive result *and* having the disease is 0.98% – so two-thirds of the people who test positive do not have the disease and do not need the drug administering to them. This would be considered to be unacceptable.

A similar scenario of false negatives and positives can be applied when looking at errors from biometric readings, for example when logging on to a computer using fingerprint technology or, more disturbingly, at an international airport checking biometric readings against security databases. False positive readings can lead to a headache for those involved, whilst false negatives could allow real criminals to slip through the net.

The statistics we use offer the chance to refine and improve upon processes that impact on our daily lives in ways we shouldn't take for granted.

what's the point of...

**PROBABILITY?**



what's the point of...

# QUADRATIC EQUATIONS?

## The Beautiful Game? Oh ballistics...

**There will always be debate about issues in football. Who scored the best goal? The best ever player? The best team?**

These debates will take up many hours and there will never be an outright winner, although Brazil and Liverpool are my choices – and I'm always right!

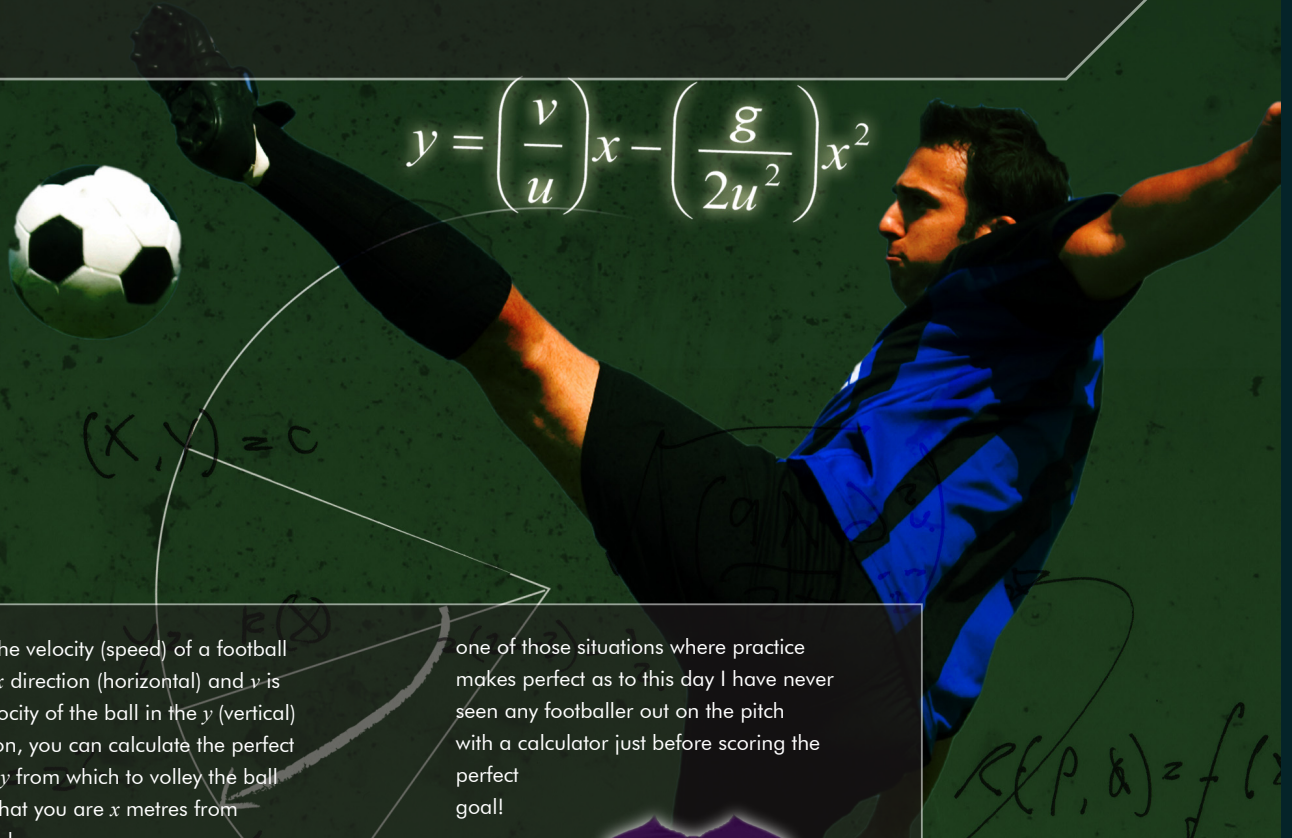
However there is no debate about one of the most technically gifted players of the modern era, Zinedine Zidane, who scored

arguably the best ever goal in 2002 in the UEFA Champions League Final.

### How did he do it?

Well, quadratic equations may help to explain the art of the volley. The principles are based on discoveries by Galileo and have many implications for military and sports enthusiasts alike.

$$y = \left(\frac{v}{u}\right)x - \left(\frac{g}{2u^2}\right)x^2$$



If  $u$  is the velocity (speed) of a football in the  $x$  direction (horizontal) and  $v$  is the velocity of the ball in the  $y$  (vertical) direction, you can calculate the perfect height  $y$  from which to volley the ball given that you are  $x$  metres from the goal.

$$y = \left(\frac{v}{u}\right)x - \left(\frac{g}{2u^2}\right)x^2$$

This includes an allowance for gravity ( $g$ ) but not for air resistance. However this is

one of those situations where practice makes perfect as to this day I have never seen any footballer out on the pitch with a calculator just before scoring the perfect goal!



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## Crime Scene Mathematics

**Quadratic equations have also been applied to the saving of lives and the analysis of crime scenes.**

When a forensics team reaches a crime scene where bullets have been fired, the application of quadratics helps to determine where a bullet was fired from.

When investigators arrive at the scene of a car crash they can work out the speed of the car at the time of the accident and make judgements on dangerous driving, etc. A car can travel from A to B by travelling at a constant speed. However in order to reach that speed it must accelerate and, using common sense, in order to stop it must decelerate (braking).

Where  $s$  is the distance travelled by a car,  $u$  is the velocity of the car,  $a$  is the acceleration and  $t$  is the time, we have a quadratic equation that links  $s$  to  $t$ .

$$s = ut + \frac{1}{2}at^2$$

If we substitute a negative value for  $a$ , then we can model deceleration and hence braking distance  $s$ . This simple equation predicts that by doubling your speed it will quadruple your stopping distance.

$$s = \frac{u^2}{2a}$$

It makes sense to drive safely – and the maths proves it ...

## Raindrops keep falling on my head... but satellites don't

**Let's perform a simple experiment. You throw a tennis ball in the air and (hopefully without hitting anyone) it should come back down having followed a parabolic path after obeying the laws of gravity. This path is essentially a quadratic equation with a negative coefficient for  $x^2$  (why?).**

In this age of rapid technological advances, we are continually and increasingly reliant on satellite technology. Without satellites there wouldn't be international mobile phone conversations, access to thousands of media channels, personal navigation systems, weather monitoring, etc. So why aren't satellites falling on our heads like the tennis ball? Think about a satellite being launched. Let's assume the Earth is stationary and completely flat along the  $x$  axis. At some point the satellite will fall back down to Earth, and this would be the range of the satellite. However the Earth is spherical, not flat, so the position of the  $x$  axis changes as we move around the Earth.

To try to understand this, draw a series of regular polygons by increasing the number of sides,  $n$ , each time by one (triangle, square, pentagon, hexagon, etc.). The more sides to the shape, the more it resembles a circle. In fact, consider a polygon with infinite sides. What shape is this? Each of the sides can be thought of as being the  $x$  axis but from a different point along the Earth's surface.

Every time the satellite reaches its range (where you expect it to land if the Earth had a flat surface), it actually hasn't. It will miss the edge because the Earth has a curved surface and so it has new  $x$  axis position. Furthermore, with the Earth actually rotating, a satellite can be launched to a precise height and speed to maintain geostationary orbit, appearing as if it were stationary above the same point on the Earth's surface, whilst actually keeping pace with the Earth's rotation. If we didn't have this, we would keep losing satellite TV feeds and end up watching less TV.

Now there's a thought ...



what's the point of...

# QUADRATIC EQUATIONS?

what's the point of...

# SEQUENCES?

## Counting the cost or splashing out?

**You struck it lucky and won £5000 in a prize draw. Having spent some of the cash on a new jacket and festival tickets, you decide to put £4000 of the money in a savings account. But which bank? And what does 5% AER mean?**

Your friend wasn't so lucky and is in debt. She owes £300 on her store card and wants to know how fast she needs to pay it off. Being able to work with sequences of numbers is vital for anyone working in the financial sector.

AER stands for the annual equivalent rate and is the percentage of your £4000 that you'll get in interest at the end of the year.

At 5% AER you'll get £200 interest after twelve months. If you've not spent any of those savings then after two years you'll have 5% of £4200, or £210 more interest. The sequence £4000, £4200, £4410, £4630.50, ... is calculated by taking each amount in turn and multiplying it by 1.05. After ten years of saving you'll have £6515.58.

How could you make more money? Use your maths skills to get a job working for the bank!

See Facts and Figures below for details of salaries you could earn using your maths skills in a bank.

Your friend can use mathematical functions found on a spreadsheet computer package like Microsoft Excel to work out what she should pay each month.



Shopping, shopping, shopping!

In most jobs a computer can do the boring bits of the calculations but you would be expected to know enough about how it works to check it's giving a good answer or to explain it to a client or colleague.

## Facts & Figures

**In 2007 the average graduate starting salary in the UK was £23 000.**

The average salary for employed people aged between 22 and 29 years was £18 000 – £19 000.

25% of employed people aged between 30 and 39 years had a salary of less than £14 500.

Graduate starting salaries in investment banks averaged at around £36 000. They hire people with 2:1s in a numerate degree like maths or science. They may also look at candidates' school performance.

## Fibonacci sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

Work out what the rule is.

What number comes next?

If you look at the ratio of one term to the previous term, this value tends to the golden

ratio:  $\frac{1 + \sqrt{5}}{2} = 1.6180339...$

21 34 55 89 144

# Compression, compression, compression



## How do you listen to music? How did your parents?

**Thirty years ago if you wanted to listen to music you had to carry around a large and heavy radio. Walkmans, the first personal stereo players, were just coming in.**

These days your iPod can fit easily in your pocket. It has more computing power than existed in the world in 1950. How do they get it so small?

Better computer memory can now hold far more data than before. Maths helps microchip designers to make microchips smaller and smaller each year.

But there's more to iPods than just the memory chips inside. A music file which takes up 10 MB of memory when stored on your hard disc can be compressed to a 1 MB file which fits better on your iPod. How does this work?

In the 1930s the American mathematician Claude Shannon invented a new science called information theory. We can understand the text message "c u l8r" even though letters are missing from all of the words. Some of the letters are redundant, and some of the letters contain the information. Redundancy is taken out in the process of compression to make files smaller. This is why the mp3 files played by an iPod are smaller: they've been compressed.

What's this got to do with sequences? Well, a sound wave can be written as a sum of different sine waves, and compression is a process that works with the sequence of these sine waves. This maths is called Fourier analysis and was invented over 200 years ago in France to investigate heat waves. Fourier analysis is widely used in many fields of science and engineering.

# Power dressing

**Stylists and designers work with shapes, colours and materials to create new fashions and update styles. Computer animation designers also need to create images, but they have to write them in mathematical language.**

A cornrow braid hairstyle depends on a geometric sequence. Geometric means that each term in the given sequence is the same multiple of the previous term. So for example  $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$  is a geometric sequence where each number is half of the previous one. You can see the braid getting smaller like this as it curls on in itself. In order to make the style fit the person, a hairdresser has to judge how much hair they use in each bit of the braid.

Hair stylists use their experience to make a hairstyle look good rather than writing down the maths. But what if you were playing a computer game where your character's hair has to move realistically? Lara Croft's ponytail swings perfectly in Tomb Raider because it's generated by a mathematical sequence. Someone's figured out the right equations to make it look real!



Cornrow braid hairstyle

## Websites to check out:

[www.mathscareers.org.uk](http://www.mathscareers.org.uk)  
[plus.maths.org](http://plus.maths.org)

Interview with two designers with a maths/science background: [plus.maths.org/issue39/interview/index.html](http://plus.maths.org/issue39/interview/index.html)

Interview with an accountant who studied maths and PE: [plus.maths.org/issue2/career/index.html](http://plus.maths.org/issue2/career/index.html)

what's the point of...

# SEQUENCES?

what's the point of...

# TRIGONOMETRY?

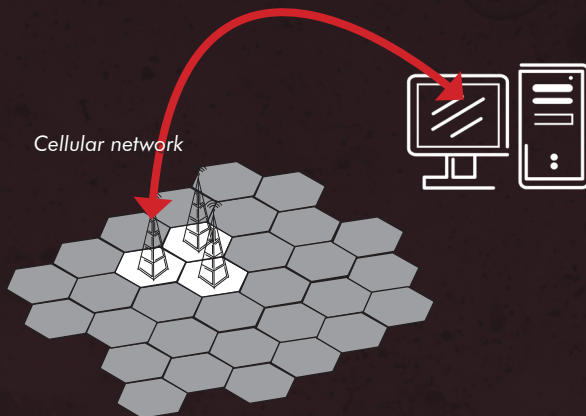
You can run...  
but you can't hide (forever...)

Going back to early July 2005, London was alive. But no sooner than the announcement for the 2012 Olympics been made, than Londoners were caught unawares by devastating acts of terrorism. The world is not always a safe place – but a little maths can help to make it a lot safer.

With security services across the world on full alert, the hunt was on for those responsible for the failed attacks of July 21st.

One of the suspects had fled to Rome in Italy. He changed the SIM card in his mobile phone thinking it would mean his phone could not be traced. However a mobile phone can be traced in two ways – using a unique identifier sent by the SIM card or by using a unique identifier sent by the handset (IMEI number).

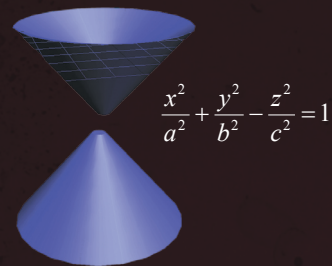
Distances and angles between transmitters on a mobile phone network can help track phone users using:

$$\frac{A}{\sin A} = \frac{B}{\sin B} = \frac{C}{\sin C}$$


Using transmitters which are positioned at known locations, the minute a call is made from a handset, it is relatively simple to work out the location of the user using the sine rule, as their location is the third point in a triangle.

Geometry and trigonometry also have important roles in civil and military applications including locating aircraft through multilateration and hyperboloid shapes. This is based on the following principle: If a signal is sent from one location then receivers in different locations will get those signals but at different times. This is very useful for tracking aircraft and satellites.

Multilateration



Is Big Brother really watching you or is the world a safer place for all the surveillance? There are some questions maths can't answer...



For further information, articles and resources visit:  
www.moremathsgrads.org.uk • www.mathscareers.org.uk  
plus.maths.org • nrich.maths.org • www.cs4fn.org

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Special thanks to Professor Peter McOwan (QMUL), Professor David Arrowsmith (QMUL), Makhan Singh, Melanie Ashfield and James Anthony, University of Birmingham

# Welcome to Hollywood

**Have you ever watched an animated movie, and thought 'how do they do that?' The chances are it is not just tracing paper and colouring pens...**

The maths learnt at GCSE and A Level can actually help bring animated movies to life.

Tony DeRose is a computer scientist at Pixar Animation Studios. He realised his love of mathematics could transfer into the real world and a really interesting job by bringing the pretend world of animation to life. "Without mathematics we wouldn't have these visually rich environments and visually rich characters," explains Tony.

Advances in maths can lead to advances in animation. Earlier maths techniques show simple, hard, plastic toys. Now, advances in maths help make more human-like characters and special effects. DeRose explains the difference a few years can make, "You didn't see any water in *Toy Story*, whereas by the time we got to *Finding Nemo*, we had the computer techniques that were needed to create all the splash effects." How do maths classes help with the animation?

Trigonometry helps rotate and move characters, algebra creates the special effects that make images shine and sparkle and calculus helps light up a scene.

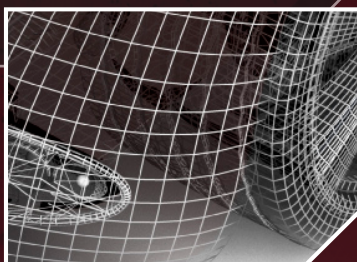
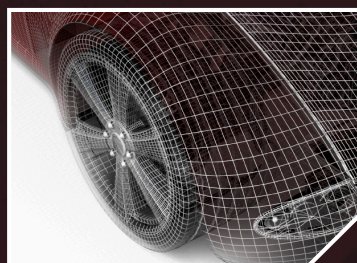
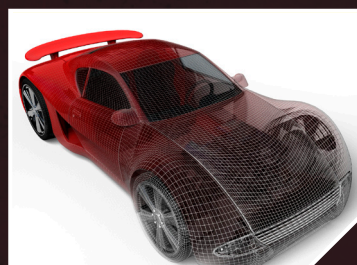


DeRose encourages people to stick with their maths classes. He says, "I remember as a mathematics student thinking, 'Well, where am I ever going to use simultaneous equations?' And I find myself using them every day, all the time now."

Even simple triangles rotating in 3D can produce results that are winning Oscars, including the manipulation of Gollum from *Lord of the Rings*.

From modern art to computer games to architecture – the humble triangle has come a long way from the text books of the ancients...

Where will your maths skills take you?



## Need a job? Know your trig!

The rough with the smooth, good times and bad times, highs and lows. There are many clichés that describe the phenomenon of the boom-slump cycle.

Did you know that using trigonometry we can forecast when there are going to be bad times and when there are going to be good times in the economy? Financial analysts and

politicians use this knowledge to plan for times of high unemployment and for making investment decisions. The peaks represent times of high employment and the troughs represent times of high unemployment.

Maths helps in planning your future. Can you plan a future without maths?

what's the point of...

# TRIGONOMETRY?