what's the point of...

INTEGRATION?

You're a Greek philosopher in the year 225 BC. What's the area of a circle with given radius?

You're a wine merchant in Austria in the year 1615. Which shape of barrels will hold the most wine?

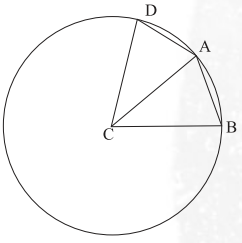
You're designing a new type of airbag to prevent head injury in car crashes in 1955. Does it work?

You're a particle physicist in 1989. How much force do you need to separate two electrons?

You need to create a better version of JPEG compression for image files. What maths will be useful?

Integration helps us to answer each of these questions. Integration is closely associated with its opposite process, differentiation. Together they are known as calculus. Related ideas have been studied for at least two thousand years. The idea of integration is based on calculating an area or volume by adding up lots of small areas or volumes that are easier to compute.

Suppose you have a circle with radius r and you've forgotten that the formula for its area is $A = \pi r^2$. You could work out the area roughly by filling the circle with triangles and calculating the area of each triangle. This is what Archimedes did over two thousand years ago to work out a better estimate for the value of π.



Give me a place to stand and I will move the earth

One of the greatest mathematicians of all time, Archimedes was born in Sicily in the Mediterranean in 287 BC and was killed in the Roman invasion in 212 BC. In between he figured out a huge amount about mathematics and physics, and designed a water pump that is still in use in Egypt today.

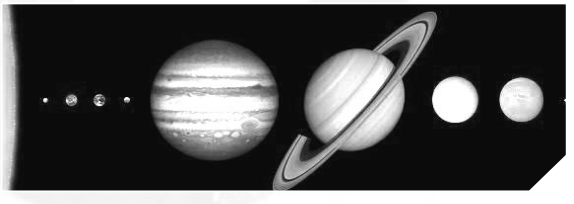
He once said to his friend King Hiero, "Give me a place to stand and I will move the earth." The king challenged him on this. Archimedes then chose a ship which needed many men to move it out of the dock, set up a pulley, and was able to move it himself without much effort.

Archimedes also showed that the exact value of π lies between the values $3\frac{10}{71}$ and $3\frac{1}{7}$ by drawing two regular polygons with 96 sides, one inside a circle with its corners on the circle (inscribed) and one outside the circle with its sides just touching the circle (circumscribed). Modern integration was born out of ideas like this.

Eighteen hundred years later...

Johannes Kepler lived in central Europe. He worked on data gathered by the Danish astronomer Tycho Brahe and figured out that the planets moved in elliptical – not circular – orbits around the sun. This is why sometimes Pluto is closer to the sun than Neptune – its orbit is more squashed.

He noticed that planets travel faster at some points on the orbit. The line joining a planet to the sun sweeps out the same area in a given interval of time, no matter where the planet is. This means that the planet must move faster when it is closer to the Sun.



At his second wedding, Kepler got distracted trying to figure out a better way to work out the volume of the wine barrels there. He wrote a book on the subject in 1615.

In both these problems Kepler used the idea of splitting up an area or volume into smaller parts in order to compute it. This is the key idea of integration.

• For further information, articles and resources visit: www.moremathsgrads.org.uk • www.mathscareers.org.uk plus.maths.org • nrich.maths.org • www.cs4fn.org

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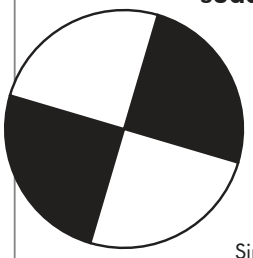
Preventing injury in crashes

You're travelling in a car along a city street at 30 mph. What happens if you have to brake suddenly?



Usually it takes 1.5 to 2 seconds to stop a car when braking normally. However in a violent impact, such as a car crash, it can take as little as 0.1 seconds to stop a car. This can cause serious head injuries.

Since the 1950s, many cars have come equiped with airbags in the dashboard. These help prevent head injuries by slowing down the deceleration of the people in the car.

In tests of airbags, a calculation is made called the Head Injury Criterion, or HIC for short. If the test gives a HIC value above 1000 then the crash would have been life-threatening. Modern cars may have HIC values of 100 to 200. The HIC is calculated by looking at every possible time interval between start time, r, and stop time, s, during the braking period and finding the average deceleration for each of those time intervals. To find the HIC we take this average deceleration raised to the power 2.5 (based on car crash data) and multiply it by the length $(s - r)$ of the time interval. The HIC is the maximum over all possible time intervals $[r; s]$.

How do you find the average deceleration? It is the integral of the deceleration, divided by the length of the time interval.

The deceleration at time t can always be found, either by integrating or by approximating the area under the curve at that point.

Now imagine the maths that Formula One engineers use to make sure their cars stay on the road even when travelling at 200 mph!

Keep it down!

There are many more applications of integration and of calculus. The JPEG 2000 image compression standard is based on wavelet theory which uses a lot of integration. Image compression ensures your photo files take less memory per image.

Calculus is needed in physics to calculate the effects of forces on tiny particles or in massive galaxies. Economists use integration techniques to model stock prices.

Integration equips you with the essential skills necessary for either a technical or scientific profession!

Websites to check out:

www.mathscareers.org.uk
plus.maths.org

Interview with maths student:
"If I've got a maths degree, I can be pretty much anything!"
http://plus.maths.org/issue28/interview/index.html

The MacTutor History of Mathematics Archive at the University of St Andrews:
turnbull.mcs.st-and.ac.uk/history/