what's the point of...

LOGARITHMS?

Disaster prevention: understanding earthquakes

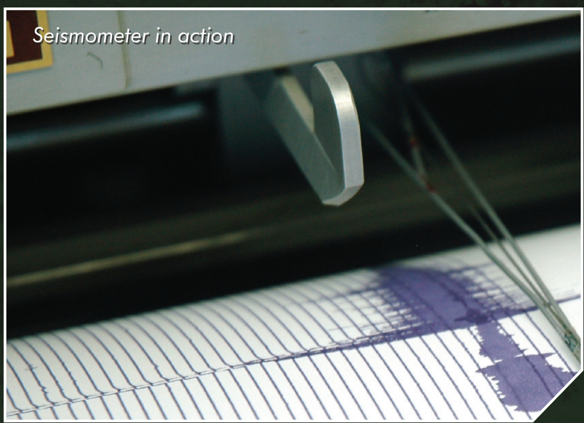
On the 8th October 2005, a major earthquake struck a mountainous region of South Asia. The shock waves radiated out from the epicentre of the earthquake, about fifty miles north-east of Islamabad, the capital of Pakistan.

It wiped out many villages and left over three million people homeless. Over seventy thousand people died in Pakistan and in the Indian-administered state of Jammu and Kashmir.

On the 23rd September 2002, a minor earthquake hit the United Kingdom. The epicentre was in Dudley in the West Midlands, north-west of Birmingham. A few homes were damaged but no-one was injured.

How much stronger was the first earthquake than the second?

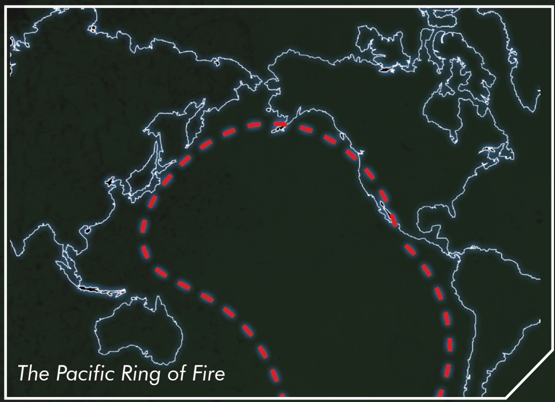
You can measure the strength of an earthquake by using a seismometer. The seismometer measures how much the earth shakes and records it as a graph. Stronger earthquakes have graphs which go up and down more: you can say that the maximum difference in height, which is called the amplitude of the graph, is bigger. This amplitude tells you how strong the earthquake is.



Seismometer in action

Where do earthquakes happen?

Nine out of ten earthquakes happen along the Pacific Ring of Fire, which circles the Pacific Ocean. Japan, California, Chile and the Philippines all lie along this ring. Seventy years ago two earthquake scientists, Charles Richter and Beno Gutenberg, were working in California. They wanted a way to tell how many of the earthquakes in California would be big ones causing serious damage. They decided to give each earthquake a magnitude number. An earthquake with a higher number would be more serious than one with a lower number. The earthquakes mentioned earlier were measured at 7.5 (South Asia) and 4.8 (UK).



The Pacific Ring of Fire

How do you calculate the magnitude of an earthquake?

These numbers are calculated by taking the amplitude of the largest wave, taking its logarithm to base 10, and then adding a factor which depends on the distance between you and where the earthquake is. Because the scale is created by taking logarithms to base 10, an earthquake with magnitude number 7 will be ten times stronger than a magnitude 6 earthquake.

How much stronger was the Asian earthquake?

We take the difference between their magnitude numbers and get 7.5 – 4.8 = 2.7. Therefore 2.7 is the logarithm to base 10 of the number we want. If we calculate 10 to the power 2.7 on a calculator we get 501.19. Try it out for yourself. This means that the Asian earthquake was five hundred times stronger than the one in the West Midlands.

Why do people use logarithms here?

It's much easier to talk about earthquakes with magnitude 6.5 or 9.0 than to talk about earthquakes with 5 000 000 or 32 000 000 000 tons of energy.

US Geological Survey's About Earthquakes page, animations, facts and photos: www.earthquake.usgs.gov/learning/

www.mathscareers.org.uk

- For further information, articles and resources visit: www.moremathsgrads.org.uk • www.mathscareers.org.uk plus.maths.org • nrich.maths.org • www.cs4fn.org

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Apple juice, coffee, milk and soap

Another scale which uses logarithms is the pH scale which measures how acidic a liquid solution is. An acid such as vinegar has a pH value of around 3.

The opposite of an acid is an alkali. Alkalis include soap and bleach. Chemically, an alkali cancels out an acid. Since many stains on clothes are acidic – tea, coffee, apple juice, milk – washing powders and bleaches are usually alkaline. Household bleach has a pH value of around 12.5.

Somewhere in between 3 and 12 on the pH scale we find solutions with a pH of 7. The pH of pure water is 7. Anything with a pH of less than 7 is called an acid; anything with a pH of more than 7 is called an alkali.

Just as for measuring earthquakes, this scale is logarithmic. This means that an acid such as lemon juice with a pH of around 2.5 is ten times more acidic than an acid such as orange juice with pH 3.5. Even your skin is slightly acidic. The soap in your bathroom probably has a pH value of between 9 and 10 so it'll help remove the sticky orange juice but won't react much with your skin. The bleach would be about a thousand times stronger, which is why you don't put it directly on your hands!

Once again, using logarithms helps us use a scale of numbers which is faster to write down.

Experiment

Get a can of cola and some dirty 1p and 2p coins. Leave the coins in a glass of cola overnight. Next morning take your coins out of the glass. The acid in the cola will make your coins look new and shiny! Why? Cola contains phosphoric acid – it's as acidic as lemon juice!

Interesting times

How much does your favourite snack cost? It probably costs a bit more than it did a few years ago. This is due to inflation – in a healthy economy prices creep up slowly. To make up for this, employers usually give their employees a cost-of-living increase in their wages each year.

What about people who save money? Banks will pay interest on your savings so that they also increase in value. They might pay it monthly, or every three months, or once a year. Which is best?

Suppose that you have £5000 in the account and the bank pays 5% annual interest, and computes it every six months. After six months you would have £5000 $\times \sqrt{1.05}$ = £5123.48. After a year you would have £5123.48 $\times \sqrt{1.05}$ = £5250.

What if banks calculated interest differently, finding the interest paid every six months by halving the annual interest rate? How much would you have after three years?

£5000.00 × (1 + 0.05 × ½) = £5125.00 after six months.
£5125.00 × (1 + 0.05 × ½) = £5253.13 after one year.
£5253.13 × (1 + 0.05 × ½) = £5384.46 after 18 months.
£5384.46 × (1 + 0.05 × ½) = £5519.06 after two years.
£5519.06 × (1 + 0.05 × ½) = £5657.04 after 30 months.
£5657.04 × (1 + 0.05 × ½) = £5798.47 after three years.

0.05 corresponds with the 5% rate. We also multiply by ½ because six months is half of a year. The interest is rounded to the nearest penny. The final amount is £5798.47. What would happen if the bank computed your interest every month, or every day?

Final amount after three years if interest is paid on £5000 or on £10 000.

INTEREST PAID EVERY:	INITIAL AMOUNT £5000	£10 000
One year	£5788.13	£11 576.25
Six months	£5798.49	£11 596.93
Three months	£5803.84	£11 607.55
Each month	£5807.54	£11 614.72
Twice a month	£5808.66	£11 616.53
Every day	£5809.11	£11 618.22
Every hour	£5809.17	£11 618.34
Every minute	£5809.17	£11 618.34
Every second	£5809.17	£11 618.34

If interest is paid more frequently, you get more. However, after a point, the extra amount gets so small as to not make a difference. Computing the interest over increasingly smaller time intervals does not result in any extra money. The maximum value you can get is the original amount multiplied by 1.161 833 7. If you take the logarithm of this to the base e (where $e = 2.718\ldots$) you get 0.15, which is 3 x 0.05 (number of years multiplied by the interest rate). This is true for any period and any interest rate. Logarithms are used a lot in investment banking for making financial calculations like this.

The number e, which equals 2.718 281 8…, is special in mathematics. It was first discovered in 1683 by Jacob Bernoulli, a Swiss mathematician who wanted to understand the compound interest problem. But it is also special because the function $y = e^x$ differentiates to itself, and for many other reasons.

Websites to check out:

www.mathscareers.org.uk
plus.maths.org
Interview with a financial engineer:
www.plus.maths.org/issue46/interview/index.html
History of the number e and of logarithms:
www-history.mcs.st-andrews.ac.uk/HistTopics/e.html

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