maths careers endless opportunities

more maths grads multiplying opportunities

what's the point of...

PROBABILITY?

Oh no...penalties...again!!!

In the summer of 2008, football fans could follow Euro 2008 without the stress of seeing any of the home nations knocked out on penalties (because they never managed to qualify in the first place).

Take England. Out of the last eight major tournaments that they have qualified for they have gone out on penalties five times (being knocked out by other means the other three times). This raises an interesting question – as the opposition manager about to play England, should you play for penalties?

In total, England have been involved in seven penalty shoot-outs in competition and have lost six of them – their only success coming against Spain in Euro '96. So is this 14% success rate statistically significant? How can England improve the odds of success in penalty competitions? Penalties are supposed to be a hit and miss affair – but with a bit of practice and some mathematical analysis, England may well overcome their penalty-taking curse.

Let's set up a simple scenario when taking a penalty.

- A striker can shoot either to his/her left or right, and similarly a goalkeeper can dive to his/her left or right.
- If the goalie dives to his/her left and the striker shoots to his/her left OR if the goalie dives right and the striker shoots right then a goal is scored (assuming the striker is accurate) because the goalie will be diving away from the ball.
- If the goalie dives to his/her left and the striker shoots to his/her right (or vice versa) then the goalie and the ball are reasonably close together and there is a 50% chance the goalie will save the ball.
- Let's assume that the striker is accurate when shooting left 70% of the time and 90% when shooting right.

Using mathematics we can estimate the best strategy for the striker to employ – it involves shooting to his/her left 56% of the time and to the right 44% of the time, irrespective of the goalkeeper's strategy. Overall this corresponds to scoring around 60% of the time. But why should the striker shoot more to his/her left side even though this is less accurate (70%) than when shooting to the right (90%)?

Using the same mathematics we can also estimate the best strategy for the goalkeeper – it suggests diving to his/her left 69% and to the right 31% of the time. So if the striker shoots to the more accurate right side, the goalkeeper will dive more often to his/her left and increase the chances of saving the shot. However if the striker shoots to the less accurate left side, the goalie will only dive in this direction (to his/her right) around 30% of the time – so the lower shot accuracy is compensated for by the fact the shot is less likely to be saved because of the goalkeeper's strategy.

(For a more in-depth perspective on the maths, please see the article by John Haigh on *Plus* magazine website: http://plus.maths.org/issue21/features/haigh/index.html)

Of course, penalties are blasted into the back of the net or accurately placed. They may be in the top left corner, straight down the middle or in the bottom right corner. The goalkeeper may elect not to dive at all or may find that reaching a penalty to the top left is more difficult than reaching a penalty aimed to the bottom left. But at this stage you simply construct a more realistic model involving more than just shooting left and right.

So practice is the better alternative, but the maths and statistics can help analyse performances. In fact, think of all the stats that underline a good performance – not just penalty taking – the distance covered by Steven Gerrard in a match, the number of tackles by Cesc Fabregas, the pass accuracy of Lionel Messi or the power of a shot by Cristiano Ronaldo – it all counts …

- Written and edited by Zia Rahman and Vivien Easson, More Maths Grads, School of Mathematical Sciences, Queen Mary, University of London (QMUL)
- Special thanks to Professor Peter McOwan (QMUL), Professor David Arrowsmith (QMUL), Makhan Singh, Melanie Ashfield and James Anthony, University of Birmingham

- For further information, articles and resources visit: www.moremathsgrads.org.uk • www.mathscareers.org.uk plus.maths.org • nrich.maths.org • www.cs4fn.org

The long arm of the law - probably

In 1999, Sally Clark was tried, convicted and sentenced to life imprisonment for the double murder of her two sons who were aged just 11 weeks and 8 weeks at the time of their deaths.

The tragedy shocked the nation, as the expert testimony of Professor Roy Meadow indicated that the chances of the double deaths happening in the same family from natural causes – Sudden Infant Death Syndrome (SIDS) commonly known as cot death – were 1 in 73 million. In other words, so unlikely that Sally Clark must be guilty of the murder of her sons.

However doubts surfaced about the testimony of the expert witness on the grounds of poor mathematical reasoning. The Clarks had always protested their innocence and there was much debate about the testimony; the Royal Statistical Society had issued a press release pointing out the mistake and indeed the conviction was quashed in 2003.

So what happened? If two events are considered to be unconnected they are said to be independent of each other. Professor Meadow made the (invalid) assumption that the two cot deaths were independent. For a non-smoking, affluent family the chance of a cot death occurring is around 1 in 8500. So to calculate the probability of two deaths occurring in one family he simply multiplied the probabilities together giving a result of 1 in 73 million. He then presented this as the probability that Sally Clark was innocent. This is a case of the Prosecutor's Fallacy. Are you guilty given the evidence or given the evidence are you guilty?

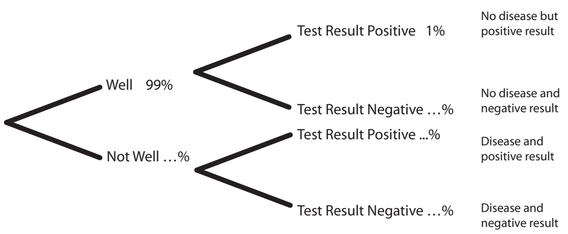
However, research suggests that in a family where one episode of cot death has occurred, the chance of it happening to another sibling is increased by between 10 to 22 times – this means that two cot deaths are certainly not independent. Also consider this, in normal circumstances the probability of either double SIDS or double murder in a single family is very small but, given that a double death has actually occurred, the chances of it being double SIDS or double murder are more likely.

Is there maths in that too? Probably

Medicines that come to the market have done so on the basis of rigorous testing and statisticians are vital to that role.

Pre-clinical trials produce masses of data that must be carefully analysed to determine safety. Clinical trials involving people can take a number of years and include the design of safe trials, the right dosage of medicine and other factors.

Suppose we undertake a screening programme to identify a disease and hence administer a cure. The aims are quite reasonable. Now suppose 1% of the group suffer from the disease and the rest are well but also that the there is a 2% chance that the test produces a false result. Using this information can you complete the following probability tree diagram?



By moving along the branches we can calculate the various probable outcomes and fill in the probabilities associated with each outcome. The two 'dodgy' outcomes are small enough to be considered acceptable. The probability of being well but having a positive test result is known as a False Positive, and the probability of having the disease but having a negative test result is known as a False Negative.

However, in real life the medication we need to administer is potent and expensive. Consider everyone with a positive test result. How many of them actually have the disease? Using the probabilities given, we see that the probability of having a positive result is 2.96% whereas the probability of having a positive result *and* having the disease is 0.98% – so two-thirds of the people who test positive do not have the disease and do not need the drug administering to them. This would be considered to be unacceptable.

A similar scenario of false negatives and positives can be applied when looking at errors from biometric readings, for example when logging on to a computer using fingerprint technology or, more disturbingly, at an international airport checking biometric readings against security databases. False positive readings can lead to a headache for those involved, whilst false negatives could allow real criminals to slip through the net.

The statistics we use offer the chance to refine and improve upon processes that impact on our daily lives in ways we shouldn't take for granted.