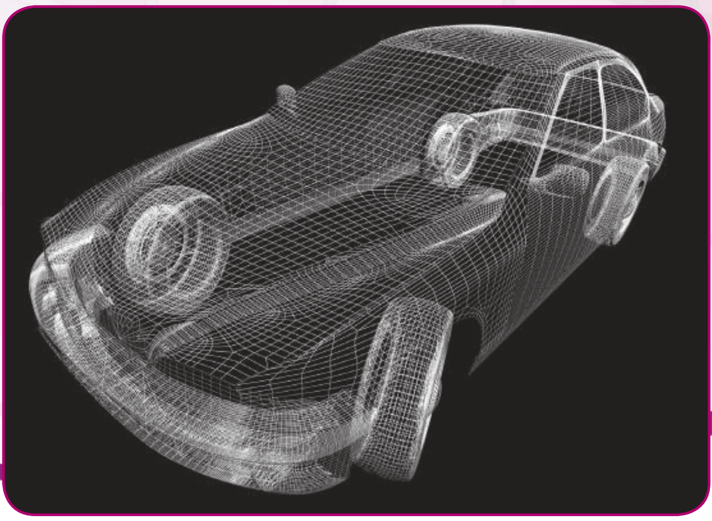
# Advancing the Digital Arts



The computer animation industry relies on a steady stream of mathematics to produce the fantastic images found on our cinema and television screens. Advanced in mathematics also fuel developments in other areas of 3D modelling, such as car design.

Duelling wizards, alien invaders and fairytale monsters have little need for proofs and equations, but advanced mathematics is essential to the computer animation and visual effects industry that creates them. The UK is a market leader in this sector, generating annual sales of around £375 million, and many of Hollywood's biggest releases are brought to life by visual effects generated in UK post-production houses. Our animators' artistic talents delight millions of viewers around the world, and their work is in turn supported by UK mathematicians.

The field of computer generated imagery has been fuelled by academic research ever since its inception in the 1960s, and as a relatively young area compared with other branches of mathematics there are still many problems to solve. In this time mathematicians have developed two key ways of representing a 3D object, each with its own strengths and weaknesses, leading to incompatibilities in the way 3D modellers work.

Those in the computer-aided design (CAD) industry, such as car designers, favour a method called NURBS, while film and video game creators in the computer animation industry prefer a method known as subdivision surfaces. Both methods allow animators and designers to build 3D models by adjusting a small number of points known as control points. For example, a CAD model of a car might use just one thousand control points to define the entire vehicle, but the underlying mathematics of NURBS transforms these points into a model detailed enough to be machined in a factory. The key difference between the two methods lies in the nature of the surfaces they produce.

> "New mathematics is pushing forward the boundaries of the animation industry."

Though both methods were developed in the 1970s, NURBS rose to prominence because computers at the time had insufficient memory to make subdivision feasible. When memory capacities finally caught up the 1990s the animation industry made the switch to the easier-to-use subdivision surfaces method, but the CAD industry continued to use NURBS because it creates smooth surfaces that can easily be manufactured. Subdivision produces surfaces that appear smooth on a monitor, which is adequate for animation, but they are not suitable for translation to the real world.

The two methods have the same underlying mathematical foundation, but over the years have been developed in different directions. Now, a team of mathematicians and computer scientists lead by Neil Dodgson at the University of Cambridge have devised a method that unifies both techniques in the same mathematical framework. Their research will eventually allow the CAD industry to use the easier tools employed by the animation industry, without discarding any of the existing models or experience built up over the past 40 years.

Dodgson's recently developed method is still at an early stage, but it provides mathematical guarantees for linking the two modelling methods that should allow the CAD industry to start incorporating it into their software. This will reduce the need for designers to learn the complicated NURBS method, and allow them to better focus on achieving the shapes they want.

His work will ultimately benefit designers in the CAD industry more than digital animators, but other kinds of new mathematics are also pushing forward the boundaries of the animation industry. One of the biggest challenges faced by animators is modelling natural phenomena such as water or plants in a way that looks realistic but can also be controlled to meet the director's requirements.

For example, a scene might call for water to flood a corridor while knocking a particular painting off the wall. Animating this is a

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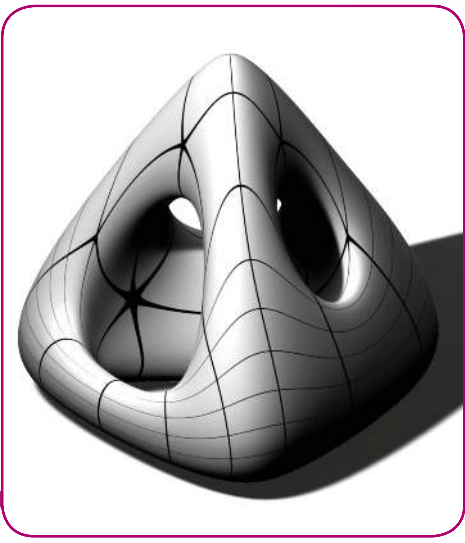
difficult problem, since the equations used by scientists to realistically model water behaviour are too slow for animators to use. Instead, mathematicians must develop simpler approximations of the advanced simulations that still give a realistic appearance but don't actually match the true physics of the situation.

This kind of problem is encountered by some of the biggest names in the business, as animators working on *Star Wars Episode I: The Phantom Menace* discovered. They were asked to animate a character performing somersaults in a long flowing robe, but when running the simulation they found that the robe would rip off and fall to the ground mid-leap. Investigating, it turned out that the character was experiencing forces similar to those felt by a fighter jet pilot, and the realistically modelled robe couldn't cope. In this case the mathematical simulation was correct, in that it produced a realistic result, but the animators had to modify the cloth robe to be unrealistically strong in order to get the effect the director wanted.



Solving these types of problems often involves complex, one-off solutions produced according to industry time frames of 6 to 24 months. Though these quick fixes get the job done, they often can't be replicated by other visual effects companies, leaving directors disappointed that they can't use similar effects to those found in the latest blockbusters. In contrast, researchers like Dodgson can investigate issues that will concern the industry in 5 to 10 years time, and by developing mutually beneficial partnerships with the industry they can provide animators with more general solutions to their problems.

Without these close ties between industry and academia, the UK may not be able to maintain its lead in visual effects. Mathematicians and computer scientists are currently developing the methods that will power the visual spectacle of the films of the next decade, bringing both financial and cultural wealth to the country. They can also apply their expertise to improving other areas of 3D modelling, as illustrated by Dodgson's research on improving NURBS. It is essential that we have a strong mathematical foundation to support the digital arts if the UK is to continue its dominance in the creative industries.

# TECHNICAL SUPPLEMENT

## NURBS

Non-uniform rational B-spline (NURBS) is the CAD industry standard for creating 3D models. Designers work with a regular rectangular grid of control points that can be arranged to represent any possible shape, then the NURBS method generates a continuous surface based on these points. Unlike subdivision surfaces, NURBS is mathematically guaranteed to produce a smooth surface, making it a suitable system for designing parts to be machined. It's also an efficient system because the computer can just store the control points and not the entire surface, but it does have some drawbacks.

One issue is the need for a rectangular grid, which means that every row in the grid must contain the same number of control points. A complicated shape such as a car cannot be modelled by a single NURBS surface because the bonnet needs a smaller grid than the roof, so designers use multiple surfaces and join them together. Stitching a NURBS model together in this way is quite a technical and time-consuming process, which is why animators prefer the more automated subdivision surfaces method. Dodgson has created a new subdivision method that is also compatible with NURBS, allowing the CAD industry to use its benefits without throwing away their existing ways of working.

## Subdivision surfaces

Like NURBS, the subdivision surfaces method for producing 3D models also uses a mesh of control points, but as these points aren't restricted to a rectangular grid they can represent complicated shapes without the need for stitching. These points are then subdivided to produce a new mesh, with additional control points determined by the position of the originals, and repeated subdivision eventually produces a smooth surface.

The computer animation industry prefers the subdivision surfaces method because it is easier for artists to use. Subdividing five or six times results in points that are closer together than the width of a pixel, so surfaces appear smooth on-screen. This level of detail isn't high enough for CAD though, which requires a much greater level of subdivision, but repeatedly subdividing can introduce imperfections in the surfaces which renders them unsuitable to be manufactured.

### References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Dodgson, N. A., Patterson, J. & Willis, P. (2010) What's up Prof?: Current Issues in the Visual Effects & Post-Production Industry. *Leonardo*, 43(1), 92-93. DOI: 10.1162/leon.2010.43.1.92



### EPSRC Grants

Reference: EP/E025889/1
Title: Non-uniform subdivision surfaces

Reference: EP/H030115/1
Title: Unifying NURBS and subdivision: Extracting sparse shape descriptions using NURBS-compatible subdivision

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