MATHEMATICS MATTERS

Building the Digital Society

Computers and networks stuffed with ever-increasing amounts of data are transforming our society, creating a digital world with its own rules and behaviours. We need mathematicians who understand this new world and can turn data into useful information for the benefit of society as a whole.

The past few decades have seen the rapid growth of digital technology in every area of life, enhancing our society in ways that were previously impossible. Now that we've all come to rely on ultra-fast communication, instant access to the world's information, online shopping and more, the nation's digital infrastructure is as essential as our waterworks or roads.

Our future digital society is still evolving, and we need to keep pace with the changes. All of the messages, interactions and transactions that pass through our communication networks leave digital footprints on databases around the world, and the next step in the digital revolution is using mathematics to transform this mass of data into useful information that aids business, government, and the public.

Digital data sets the new society apart from the old, and it comes in many forms. Social networks, supermarket loyalty cards and energy smart meters are just some of the mechanisms for capturing vast amounts of data, but turning this data into information involves more than just recording it. Mathematicians such as Peter Grindrod at the University of Reading are currently developing a new mathematical discipline to handle and interpret this wealth of data, and while it is currently in an early phase, the study of mathematics for the digital society is expected to be commonplace within the next ten years.

Britain has the opportunity to become a world leader in this field, building on our existing successes to create a new understanding of digital interactions. This expertise can then be exported all over the world, providing valuable insight to international businesses and organisations, while helping to secure Britain's position as a knowledge-based economy. Without essential mathematics research into modelling the digital economy we run the risk of being left behind by other nations such as the United States, but with the help of new mathematical insight, Britain can lead the way.

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Modelling the digital world is not like modelling the physical world, where established equations govern the movement of atoms or the flow of electrons. Interactions between people and information are more complicated, and we need to develop new concepts and models to understand and predict their behaviour in the new digital society.

Networks are a key component of this new society, as illustrated by the rise of mobile phones, email, and social networking websites, yet mathematicians still don't fully understand how networks evolve. Most mathematical models of networks are static, dealing only with fixed links between certain points, but the networks in the digital world are constantly changing, and Grindrod's research aims to take these effects into account.

Suppose we're interested in how a group of people communicate through a particular medium, such as email. We can represent each person as a circle, and the emails they send as lines connecting the circles. This collection of lines and circles is a mathematical network, and as it evolves the circles remain the same, but the lines appear and disappear as emails are sent back and forth. Some lines might always be present, representing constant communication, while others might never appear, representing two people who aren't in contact because they don't know each other's email addresses.

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Studying evolving networks lets us see how different people use communication tools. Some individuals might be good at broadcasting information, reaching out to lots of circles, while others might be good listeners, receiving messages from a variety of sources. Network operators could also monitor for any patterns that don't match the model's predictions, which could indicate a traffic spike or unusual behaviour. These theories could be applied to network management, surveillance, or even viral marketing.

It's not just communication networks that can provide useful information though. The data collected by businesses during online transactions, banking, and supermarket shopping can potentially offer new insight into their customers' behaviour, but further mathematical research is needed to help unlock its full potential.

Businesses already use this data to segment their customers into different groups, but as with networks, these models are static. A bank might place customers in a particular group based on their age or location and target them with certain products, but these segmentation models are inflexible and can't take into account a customer's changing behaviour. A more intelligent model could learn over time, shifting customers into different groups as the bank builds up their data profile and ultimately leading to a more successful businesses.

Accurately predicting a customer's behaviour will also allow businesses to target them with appropriate messages. Rather than the blanket mail-drops or emails we have now, each person could receive a message specifically designed to appeal to them. As a result, the customer is less likely to be annoyed or discard the message as spam, and more likely to purchase products and services from the business.

This kind of customer analysis has grown over the last twenty years in the retail and banking industries, but it's now being adopted in other sectors as usable data becomes available. The roll-out of smart meters across the UK will for the first time give energy providers real-time details of individual household consumption, allowing them to manage and predict demand based on customer behaviour. This will require a new mathematical understanding of how electricity is used, especially in a world of electric vehicles, micro-generation and renewable energy, all of which introduce new complexities into the power networks.

All of these examples are enabled by advances in ICT and our digital infrastructure, but they won't be possible without the mathematics to support them. Technology has always transformed society, from the industrial revolution of the 18th and 19th centuries, through the rise of physics during the Second World War, to recent developments in biotechnology, but each shift has required a new mathematical understanding to model the processes and concepts behind the change, and our current revolution is no different. A modern digital society needs mathematicians to transform data into information, so that we can all reap the benefits.

TECHNICAL SUPPLEMENT

Evolving networks

Mathematicians have long used sets of edges and vertices, known as graphs, to depict the relationships between different objects in a network, but these graphs don't capture the behaviour of networks that change over time. To solve this problem, Peter Grindrod's research models networks with evolving graphs, a sequence of graphs that changes in a probabilistic way. In this model the connecting edges between vertices are either "alive" or "dead", indicating whether communication between the two vertices is taking place, and at each point in time a new edge can be "born" or an old one can "die". Edges can also be "immortal" or "extinct", reflecting constant and no communication, respectively.

This model displays four long-term behaviours, depending on the rate of edge births and deaths. The resulting network graph can be one in which each vertex is linked to all the others, or one in which there are no links at all. Alternatively, each edge in the graph can become either immortal or extinct, leading to a static graph, or the edges can continue to change, leading to a graph of random connections.

Applying the model to real-world data from a Chinese social network shows that the longer two vertices remain disconnected, the less likely it is that they will connect in the next week. In other words, if two individuals don't make a connection when they first join the network, it becomes increasingly unlikely that they ever will as time goes by.

References

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