MATHEMATICS
MATTERS

Danger: Rogue Waves

Rogue waves appear without warning, towering high over ships and oil rigs. Traditional mathematical models couldn't predict the occurrence of these dangerous waves, but the latest techniques let oceanographers make accurate forecasts. The research helps to protect our trade, energy and food supply routes.

Sailors have always told tales of rogue waves, describing encounters with gigantic walls of water that appear without warning and drag ships to the depths below. Until recently, oceanographers dismissed these accounts as nonsense, no more real than sea monsters or mermaids, because mathematical models predicted that such an immense wave was almost impossible. That all changed on New Year's Day 1995, when measuring instruments aboard the Draupner oil platform in the North Sea detected a wave nearly 26 metres high: the first verifiable rogue wave.

Mathematicians have studied waves for centuries but this incident made them realise that more research was needed, as it demonstrated that rogue waves can occur more often than traditional mathematical models would predict. Now, the development of new models has given us a greater understanding of what causes rogue waves and how to avoid them.

A rogue wave may sound similar to a tsunami, but there are fundamental differences. Tsunamis are caused by massive disturbances in the ocean, such as a volcano or an earthquake, creating an upheaval of water that becomes a devastating wave as it reaches the coast. It is difficult to anticipate the event that causes a tsunami, but once it occurs oceanographers can use mathematical models to predict where the effects will be felt.

In contrast, rogue waves appear suddenly in the middle of the ocean, seemingly without cause, and disappear just as quickly. They can't be predicted and their transient nature makes them difficult to observe. To combat this constantly changing environment oceanographers measure wave heights with statistics, rather than individual observations. A commonly used statistic is the significant wave height, or the average height of the highest one-third of waves. The significant wave height already corresponds to a larger-than-normal wave, but a rogue wave is twice as high as this or more. At the time of the Draupner incident, the significant wave height was around 11 metres, far below the level of the 26 metre rogue wave.

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Waves this large were unexpected because traditional wave models are linear, meaning that two waves added together will make a third that is at most twice the height of the originals. This model works well for many kinds of wave interactions, but even before the Draupner incident mathematicians had discovered that not all waves could be explained by a linear model.

In the 1960s, the English mathematicians Thomas Brooke Benjamin and Jim Feir found that waves of constant height would become unstable over time. Some sections of the wave would grow deeper as others became shallower, resulting in a temporary wave that was much higher than those around it. They realised the waves were interacting in a nonlinear fashion, with one wave "borrowing" energy from the others to become up to five times larger.

A rogue wave works in much the same way, and it turns out that both kinds of wave can only be explained by a model called the nonlinear Schrödinger (NLS) equation. It is related to a linear equation that forms the basis of quantum mechanics, but was extended to cover nonlinear water waves by the Russian physicist Vladimir Zakharov in 1968, and is now an essential tool for studying waves in many branches of physics.

The sheer size of the ocean makes it impossible to model individual waves, so oceanographers divide the sea into

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regions of 100 km² and use the NLS equation to predict the average sea state within each. By feeding observational data into the equation they are able to produce statistical measures that describe the general behaviour of the sea surface at a given point in time, such as the significant wave height discussed above. These measures form a probability distribution that describes how often waves of a certain height are likely to occur in a region, and this information allows shipping companies to avoid dangerous areas of the sea when plotting their routes.

It is difficult to estimate the effect this has on ship survival rates, but routes plotted with the help of this information increase a ship's efficiency by roughly 5%. Given that the daily fuel consumption of a typical ship costs around $55,000 (£35,000), this translates to an average saving of over $2,000 (£1,300) per day for each vessel.

This system has been in place for the last 20 years or so, but mathematicians are constantly finding new ways to improve its accuracy. A recent development was the Benjamin-Feir index (BFI), which was introduced in 2003 by Peter Janssen, a mathematician at the European Centre for Medium-Range Weather Forecasts (ECMWF) in Reading. The BFI measures the level of nonlinearity in a given sea region, with higher levels of nonlinearity indicating a greater chance of rogue waves. Janssen showed how to calculate the BFI from observational data and the NLS equation, allowing for a more accurate prediction of rogue waves than previous methods.

The BFI is just one of many improvements that have been made in wave prediction since the early 90s. Mathematicians have worked with oceanographers to lower the average error in predicted wave heights from 30% to 10%, reducing uncertainty and allowing shipping companies to handle unexpected events more easily. The work isn't complete however, as the NLS equation only considers waves that travel in one direction, and extending it to cover multiple directions at once is an active area of mathematical research.

Our understanding of waves has come a long way since the days when sailors still believed in sea monsters, and this progress is due to fundamental mathematical research into the equations which govern waves on the ocean surface. As an island nation, the UK relies on the sea for much of its trade, energy and food, so it's essential that ships are able to pass safely across the ocean without encountering rough conditions or devastating rogue waves. Mathematicians have helped secure this safety for the past twenty years and will continue improving their methods to provide even better predictions, saving both money and lives as a result.

TECHNICAL SUPPLEMENT

Benjamin-Feir Index

Simple linear models of ocean-wave height produce a Gaussian or normal distribution, the familiar "bell-shaped" curve found in all areas of science. This seems like a reasonable measure of wave-height probability, because we might intuitively expect waves of average height to occur most often and extreme events such as rogue waves to occur less often, but nonlinear effects mean that ocean waves can deviate considerably from the Gaussian distribution. The Benjamin-Feir index (BFI) is a way of describing this deviation, allowing oceanographers to calculate more accurate probabilities of rogue waves occurring.

The BFI is equal to the ratio of the steepness of the waves and the width of the frequency spectrum, and determines the kurtosis, or deviation from the normal, of the wave height distribution. If the spectrum of a particular sea state is narrow and the waves are steep, the BFI is large and deviations from the normal distribution are more likely. Conversely, if the spectrum is wide and the waves are shallow, the BFI is small and rogue waves are unlikely to occur.

References

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