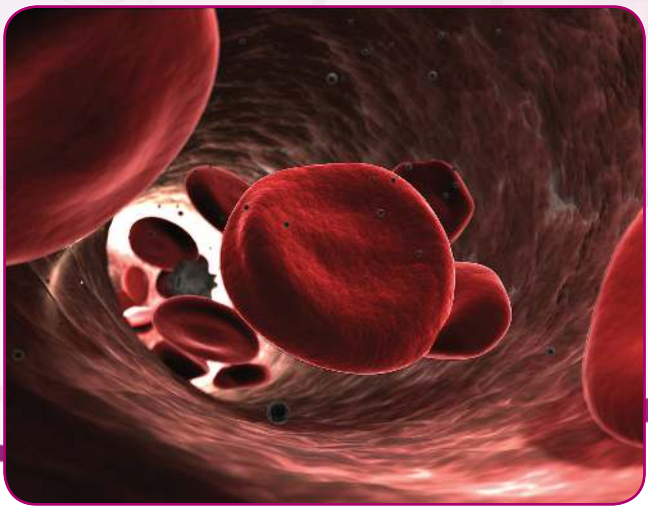
Keeping Hearts Pumping



Blood-related diseases can seriously harm patients' quality of life and even lead to death. Many of these diseases are caused by problems with the flow of blood in the body, and using mathematical models to understand how and why these occur can help save lives.

Our bodies contain thousands of miles of blood vessels, from millions of tiny capillaries each ten times thinner than a human hair, to the 3 cm-wide aorta connecting the heart to the lower body. The passage of blood through this dense circulatory system is a complex process, but one that mathematical modelling can make easier to understand. Learning more about how our blood flows and what can cause flows to change is an important part of understanding many blood-related diseases.

Heart disease remains one of the UK's leading causes of death, with nearly 100,000 cases each year. A further estimated 2.6 million people in the country currently live with the condition, despite efforts to improve public health by reducing levels of cigarette smoking and increasing awareness of the benefits of a good diet and regular exercise.

Poor diet and lack of exercise can also lead to hardening of the arteries, as fatty deposits known as plaques slowly accumulate in the walls of diseased blood vessels. A plaque large enough to impede blood flow in a vessel supplying oxygen to the heart muscle can result in a heart attack, and if a plaque breaks free from a diseased vessel it can be carried away and lodge in the brain, where the resulting oxygen deprivation can lead to a stroke or even death.

Mathematical explanations for blood flow date back to the 1840s, when the French physiologist Jean-Louis-Marie Poiseuille discovered that the flow of liquid in a tube can be described using a simple mathematical formula. Poiseuille's equation relates the rate at which a fluid flows through a tube to the difference in the pressure at either end. Although Poiseuille's original intention was to explain how pressure affects blood flow, his equation works for almost any liquid in a tube and plays an important role in modern fluid mechanics.

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Around the same time, the French physicist Claude-Louis Navier and the Cambridge mathematician George Gabriel Stokes derived more general equations for describing fluid flow. Their Navier-Stokes equations are simple to write down but extremely difficult to solve directly. Instead, mathematicians and engineers such as Oliver Jensen at the University of Nottingham use sophisticated mathematical and computational tools to obtain solutions to many important fluid-mechanical problems, including blood flow.

Blood vessels follow complex paths through our bodies, twisting, turning and branching to ensure efficient delivery of oxygen from head to toe. The pattern of flows within our blood vessels depends on their size and shape, and different flow behaviours are found in the large arteries and veins compared with those in the capillaries.

The differences are due in part to the size of our red blood cells, which can only just squeeze through the smallest capillaries in single file. The arrangement of individual red blood cells in capillaries is important in ensuring that the viscosity of blood is effectively smaller in capillaries than it is in large vessels, a phenomenon known as the Fåhraeus-Lindqvist effect that can be modelled with a modified form of Poiseuille's equation. In contrast, blood cells within large blood vessels follow the normal mathematical model for fluid flow as laid down by Navier and Stokes, but the intricate geometry and flexibility of the vessels creates complex flow patterns that are difficult to model.

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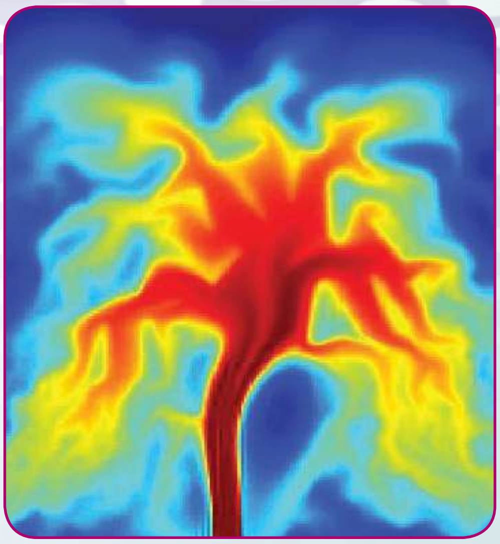
Modelling the various flows within our blood vessels is important because the cells that line the vessels, known as endothelial cells, can sense and respond to changes. If endothelial cells are exposed to abnormal patterns of blood flow, they can initiate a sequence of events in the vessel wall that ultimately lead to the formation of fatty plaques. Using flow simulations helps us to model these flow patterns and link them to sites of disease, identifying the "geometric risk factors" - particular points where blood vessel geometry could lead to the build up of plaque.

Modern medical imaging technologies now allow this modelling to take a personal approach. An angiogram works by injecting a contact medium into the blood vessels and capturing their layout with an X-ray. This and other methods, notably magnetic resonance imaging, provide detailed pictures of an individual's particular blood vessel configuration and can be used in computing their blood flow pattern, providing valuable insights to clinicians in planning appropriate therapy. This technology is still in development, and mathematical simulation is helping to work out the details.

Simulations aren't just useful for predicting flow patterns in individual veins and arteries though, and it is actually possible to build mathematical models of all the different components of the cardiovascular system. Jensen and other researchers are currently examining how to link these models together to create integrated models of both blood flow and the biological functions of the surrounding cells, organs and tissues across the whole body. These collective models will provide a predictive tool to assess a variety of situations, such as how a drug that acts on heart muscles might also affect the delivery of oxygen to the brain. It is a complex task that will take some time to achieve, but the potential benefits that such advanced mathematical models can bring are enormous.



TECHNICAL SUPPLEMENT

Blood flow and the Reynolds number

The flow patterns of blood within veins and arteries vary because of changes in the relative size of blood vessels and the speed of the flow within them. An important factor in modelling these differences is the Reynolds number, the ratio between the resistance of liquid to movement due to inertia and the resistance due to viscosity.

The Reynolds number in smaller blood vessels is low, meaning that blood flows smoothly in what is called laminar flow. In contrast, the Reynolds number for large blood vessels is high, and the flow of blood is disordered and more difficult to model.

Navier-Stokes equations

The Navier-Stokes equations are derived from Newton's second law of motion, force equals mass times acceleration, and describe the relationship between the velocity, pressure, viscosity and density of a moving fluid. As a linked set of nonlinear partial differential equations they can only be solved analytically in a few very simple cases, hence the need for the computational methods used by Jensen and others.

These methods allow us to apply the Navier-Stokes equations to a range of practical situations, but mathematicians aren't yet able to fully explain how the equations work, particularly in the case of turbulent flows that arise at very high Reynolds numbers. The problem is considered so important that the Clay Mathematics Institute has offered a $1 million prize for a proof that furthers our understanding.

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