MATHEMATICS MATTERS

Marine power: Britannia rules the waves

With an increased appetite for renewable energy sources, eyes are turning towards the abundance of water surrounding the British Isles. Harnessing marine power effectively requires detailed mathematical analysis.

Britain has a long and illustrious relationship with the sea. From the Spanish Armada and the Battle of Trafalgar, to Captain Cook's map-changing voyages around Australia and the merchants of the East India Company, Britannia is synonymous with the waves. But, nowadays, Britain has moved away from empire and exploration and is instead leading the maritime world in another way: by harnessing the power locked up in waves.

The UK is out in front as the leader in marine energy: we have more devices designed to unlock the energy of the sea than the rest of the world combined. Along with wave power, marine energy also includes exploiting the tides. As a clean, green source of power with much untapped potential, the marine energy industry is forecast to be worth £50 billion by 2050 and could employ 20,000 workers as soon as 2035. The wave energy around Britain today alone could, theoretically, supply three times our current electricity demands. The trick is converting that energy into electricity. It is not possible to capture all of the energy, but estimates suggest that in the future wave power could contribute as much as a quarter of our electricity needs.

So how does it work? Waves are generated by the transfer of energy from wind to water. The stronger the wind, the longer the distance the waves travel and the higher they are. This is important because the energy in a wave is related to the height of the wave squared (multiplied by itself). So a two metre wave contains four times the energy of a wave one metre high. There are various methods of extracting this energy.

> “The marine energy industry is forecast to be worth £50 billion by 2050”

One way, favoured by UK company Pelamis Wave Power, is to use a device that looks like a giant sea snake. About the length of five train carriages, and installed between 2 and 10 kilometres from the shore, it is made up of several hinged sections. Installed so the waves hit it head on, the moving water causes Pelamis's hinges to bend up and down. This action moves hydraulic rams which drive an electrical generator. The electricity is then ferried to land via an underwater cable. Pelamis have produced six full scale machines to date, each capable of satisfying the electricity needs of around 500 homes. Their vision for the future is commercial-scale offshore “wave farms” where many Pelamis snakes work simultaneously to add power to the grid.

To be successful, however, any wave power extraction technique has to strike a balance between calm and stormy seas. During mild conditions the waves are relatively small and so you want to get as much energy out in the most cost effective way. However, you also want your device to withstand the tumultuous conditions when conditions take a turn for the worse. To achieve this the movement of the machine is set to respond in time with the incoming waves during calm seas. It rocks in time with the sea, an effect known as resonance. Crucially, however, the default state of the machine is not resonant, allowing it to cope with choppy waters.

Designers obviously want to capture as much as the wave energy as possible and mathematical modelling has been used to work out the best design for wave energy converters. The energy liberated is often referred to in terms of the ‘capture width’ – the dimension of a wave front from which you can get 100% of the energy out. This in turn depends on the waves created by the machine itself as it bobs up and down in the water. If travelling against the oncoming wave, these waves work to cancel it out and the amount of energy available is reduced. This clearly isn't desirable.

With a single converter – known as a monopole - the wave pattern created moves outwards in all directions, similar to the ripples created by dropping a stone into a

Institute of mathematics & its applications

pond. However, mathematical analysis shows a set of monopoles arranged in a line only creates additional waves 'behind' it – that is in the same direction as in the incoming wave. Designs of this form can theoretically extract almost five times as much energy as a monopole device because they are not working against the natural waves. The mathematics shows that achieving that theoretical limit requires the length of the 'line converter' to be twice that of the incident wavelength – the distance over which the wave pattern repeats itself. The current Pelamis snakes are the length of a single wavelength, extracting three times as much energy as a single monopole.

With focus increasingly turning towards renewable energy as a clean and green alternative to dwindling fossil fuel supplies, wave power is particularly attractive for an island nation like the UK. The industry is, however, a challenging one, and it is mathematics that is helping to make strides toward the goal of part-powering our country using the sea.

TECHNICAL SUPPLEMENT

The 'fundamental theorem of wave power' shows how the wave power absorbed by a wave energy converter (WEC) is related to the water density, acceleration due to gravity, wave frequency and, crucially, the wave number (k) which is equal to 2ϖ divided by the wavelength (λ). Analysis shows that for monopole and dipole WECs the capture width is limited to $1/k$ and $2/k$ respectively, or written in terms of wavelength, $\lambda/2\varpi$ and $2\lambda/2\varpi$. This limit does not apply to a 'line converter' of many monopoles and so arrays such as these are the currently favoured method for economical wave power extraction.

References

Berrou, C., Glavieux, A. & Thitimajshima, P. Rainey, R.C.T. (2012). Key features of wave energy. *Phil. Trans. R. Soc. A*. 370, 425–438.

Stansell, P. & Pizer, D.J. (2012) Maximum wave-power absorption by attenuating line absorbers under volume constraints, ***http://arxiv.org/abs/1112.3494***

The IMA would like to thank Rod Rainey, Atkins Global, for his help in the preparation of this document.