Mathematical modelling: simulation for soldiers

The modern military is equipped with many tools, gadgets and weapons. However, mathematics is a powerful tool too as it helps simulate training environments to keep the military at the top of their game.

Modern warfare has never been more technological or more intricate. The average soldier carries equipment the cost of which runs into five figures. They operate machinery running to many times more. But with increased complexity comes an increased need for training, either on new equipment or for new theatres of warfare.

That's where simulation can play a big role and it is increasingly supplementing, or in some cases supplanting, live training. One estimate suggests the annual global spending on military simulation, modelling and virtual training will nudge $10 billion in the next decade. It costs around £30,000 an hour to train someone in a top of the range military jet, for example. An effective flight simulator can do the same job at an hourly cost of just £3,000. With flight training typically lasting one hundred hours, the financial incentive is clear. Not the mention the cost of potential damage – it costs nothing to crash or break a plane on a simulator. Then there is the ability to tailor the training to a variety of different situations and territories without ever needing to set foot in them.

Such simulation is not restricted to flight training, of course. Each training system can be tailored for specific environments including both military and civilian applications, from training sailors in maritime security to teaching miners about underground safety. Throughout all of these disciplines mathematics has a large role to play from the design of the simulation to its execution and subsequent analysis.

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Before anyone can use a simulator to fly a plane, navigate an unusual territory or walk around a mine, the environment around them has to be constructed. For overseas arenas of war this environment can be built from satellite imagery. However, this only yields a two dimensional view of the area. To be useful in a simulation this has to be adapted into a 3D replica. Sometimes this can be as simple as using the length of shadows and time of day to give height, just as Galileo did to work out the height of lunar mountains in the 17th century. NASA radar maps are also used to this end. Using this information to yield a 3D rendering of an environment requires the mathematics of co-ordinate transformation geometry, mapping two dimensional co-ordinates onto a 3D surface and adding in the additional information. Even removing the clouds in satellite imagery needs the mathematical algorithms bound up in image processing.

Once the 3D environment is built, the equipment needs to be placed within it. Not only that, the laws of physics need to be recreated to ensure the equipment responds to its surroundings as realistically as possible. A laser beam needs converting into a bullet trajectory; forces and moments for rotors, fuselages and undercarriages need to be accurate. If moving through water or air, fluid dynamics becomes important and that involves the mathematics of differential equations – particularly the Navier-Stokes equations. The simulation also has to react in real-time to the input of the trainee or trainer, often at sixty frames per second, so the software needs to constantly churn equations on the fly.

To ensure the accuracy in performance of simulated military platforms such as planes, tanks or ships, simulators use data from loggers deployed on real-life missions. The challenge comes in turning this colossus of data into something useful for the simulation. Simulation engineers use data mining algorithms to draw out patterns in the data to achieve this. These patterns can also be used to fill in any gaps in the data with sensible values.

With the simulated environment built and the simulated equipment placed within it, both need to be incorporated into a training environment. Trigonometry – the mathematics of lengths and angles – is important here to ensure that screens, sounds and viewpoints are all placed in the appropriate places relative to the user. If

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trainees need to move around within the simulation – say a battlefield environment – they need to be tracked in three dimensions. In simulations of underground mines, for example, Nintendo Wii remotes are used as torches. Both require real-time triangulation from several motion sensors, as well as analysis of the interference between them.

Throughout the training exercise a vast amount of data is collected on the performance of each candidate and this can be fed back to trainers and mathematical analysis of this data can show particular areas of strength or weakness.

Warfare has never been so advanced and so reliant on a swathe of high tech gadgetry, meaning appropriate training has never been more vital or time intensive. However, the same strides in technology mean that military personnel can train for far-off terrain from the comfort of home territory. But without mathematics' indispensable contribution throughout the process, from design, implementation and analysis, it simply wouldn't be possible.

TECHNICAL SUPPLEMENT

Co-ordinate transform geometry

Geometric shapes can be transformed using techniques including translation, rotation, scaling and skewing. If the transformations are linear they can be represented as invertible (non-singular) matrices. Matrix algebra can then be used to calculate the resulting transformations.

Navier-Stokes equations

The Navier-Stokes equations are a set of non-linear partial differential equations derived from Newton's Second Law of Motion – the one that relates force to changes in motion. The fluid environment – say the air around a fighter jet – is divided up into grids. Each grid is assigned three components of velocity based on factors including weather conditions and flying altitude.

The equations can then resolve the flow of the atmosphere, suggesting how conditions will change over a small amount of time. Those new conditions are then put back into the equations for another short period of time, and so on, to update the solution. That allows a simulation to respond in what is effectively real-time.

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