Quieter skies: reducing aircraft noise

Residents of towns under the flight paths of major airports can be plagued by intrusive aircraft noise. However, mathematics is helping quell the noise through better engine design, enabling airlines to stick to ongoing noise reduction targets.

The UK is home to Europe's noisiest airport. Heathrow has picked up this unwanted accolade thanks in part to the sheer number of people living under its flight path. Those residents of south-west London are regularly exposed to jet noise in excess of 57 decibels. With the number of flights set to almost double by 2050, residents and jet manufacturers alike are keen to reduce the noise generated by aircraft, not just at Heathrow but across Britain and around the world.

Regulating aircraft noise is nothing new - it has been around since the 1960s and 1970s. However, the restrictions placed on engine manufacturers are becoming more stringent. Those needing to meet these tough targets are turning to mathematics. The approach is two-fold: to reduce the noise generated by the engine in the first place and to limit the propagation of that noise to the ground. Building and trialling many different engines is prohibitively time consuming and expensive, and that's where numbers and equations can help.

Engineers use something called the Lighthill Acoustic Analogy (LAA), named after British mathematician Sir Michael James Lighthill (who incidentally also founded the IMA in 1964). The LAA reformulates a famous set of equations - called the Navier-Stokes equations - which are ubiquitous when dealing with the flow of fluids like air and water. This allows them to model the acoustic field around the blades that rotate in the engine to produce thrust.

"Whether it is in skies above the UK or in the waters surrounding our islands, mathematics is being employed to tackle the problem of noise"

As the plane flies, the blades and the air move at different speeds. However, at the point where the air touches a blade, they are forced to have the same speed. This is known as a "no slip boundary condition". This difference in speed between the air directly in contact with the blade and the air further afield creates a thin region around the blade, called a boundary layer, where viscous effects can drive turbulent whirlpools in the fluid called eddies. This generates noise, although, for the most part, at a fairly low level. However, when the flow of air reaches the trailing edge of the blade, that no slip boundary condition disappears and the boundary layer is left behind. This amplifies the noise because the sources of the sound lose some of their ability to cancel each other out, and the sound transmitted to a distant listener is amplified by scattering from the blade-edge. There is a cascade effect as the noise scatters off the other blades.

In tackling the problem, engineers are able to use mathematics to model the effect that altering the design of the blades would have on the noise produced. Within the model they can virtually change the angle of the blades, for example. They can also weigh up the benefits of blunt versus rounded blade edges. There has to be a careful balance, however, between the desire to reduce noise and the need to achieve the optimum amount of motive force and lift to power the plane in the first place. The financial incentive for achieving a healthy compromise is clear: at East Midlands Airport, for example, any airline caught exceeding pre-defined noise limits is fined £750 for the first decibel, then £150 for each subsequent decibel.

However, it is not just in the air where the mathematics of noise reduction is playing its part: it is also being used for underwater vessels. Underwater noise affects the behaviour of dolphins and whales. The basic mathematics is largely the same, however the sea provides its own set of challenges.

For one, the speed of sound is significantly

Institute of mathematics & its applications

greater in water as the molecules that help transmit the sound are more closely packed in a liquid than in a gas like air. Rotor blades also turn slower because of increased resistance from the water. As with aircraft, there is also a trade-off between "near-field" and "far-field" noise. There is no need to limit noise that doesn't travel very far from the vehicle as it will not reach a listener. And, once again, it is much faster and cheaper to isolate the best compromise using mathematical modelling than to experimentally test many configurations.

One way to do this is to use a technique called Large Eddy Simulation (LES), first proposed in 1963 by American meteorologist Joseph Smagorinsky. Normally, the Navier-Stokes equations are far too complex to solve directly for any given scenario. However, as its name suggests, the LES method focuses only on the large eddies – discarding the smaller ones saves precious computing time by reducing the number of computational steps required, meaning that modern supercomputers are able to tackle the problem.

Whether it is in skies above the UK or in the waters surrounding our islands, mathematics is being employed to tackle the problem of noise. With the amount of traffic set to rapidly increase as the world becomes increasingly global, those numbers and equations have never been more important.

TECHNICAL SUPPLEMENT

Boundary layer

Where the fluid is constrained to have the same speed as the solid surface (a no slip boundary condition), it creates a pressure gradient and viscous effects are driven in this boundary layer creating eddies. The thickness of this boundary layer depends on a dimensionless quantity called the Reynold's number. Although named after Osborne Reynolds, who brought it to wider attention, it was initially formulated by George Gabriel Stokes - the same mathematician jointly responsible for the Navier-Stokes equations. The Reynold's number is the ratio of inertial to viscous forces and factors in the velocity, distance of travel and kinematic viscosity of the fluid.

Amplification

Once the fluid reaches the trailing edge of the blade the no slip boundary condition disappears and noise becomes amplified. This is because the sources of the sound change from quadrupole to dipole sources. A quadrupole sources consists of two dipoles with opposite phase. There is no net force on the fluid and so it is the fluctuating stress on the fluid that creates the sound. As fluids aren't proficient at supporting shear stress, quadrupoles radiate sound poorly. However, once the source of the sound becomes dipole, there is a net force of the fluid and so the sound is able to radiate more efficiently. So engineers use mathematics in order to alter the blade shape in order to minimise this effect.

The IMA would like to thank Dr. Philip Cotterill (Thales and The University of Manchester) for his help in the preparation of this document.