Networking for the Future

Modern society relies heavily on a variety of networks, but we don't fully understand how they behave. Mathematical network theory lets us create models of our communication and transport networks, revealing new patterns and insights that will improve network capacity, reliability, and efficiency.

Networks are all about making connections. The global internet and telephony networks form the largest and most complex machine the human race has ever constructed, allowing billions of people to exchange vital information in the blink of an eye. Equally important is the global transport network, which enables the mass movement of people and goods by land, air and sea. Understanding how these connections work is essential if we want to improve them, and only the mathematics of network theory can get the job done.

A problem common to both communications and transport is the balancing of an individual's needs with the needs of the whole network, a principle that was formalised by the English transport analyst John Wardrop in 1952. Individual car drivers pick the shortest routes to their destination based on personal knowledge of the network (road paths, average traffic levels, etc.), but these individual choices can actually cause the network to be less efficient overall. A similar problem arises in computer networking, with local routers having to decide which paths to direct individual data packets along.

If every driver takes the shortest path to their destination they could collectively cause a traffic jam as the road gets overloaded, but if enough drivers are sent along alternative paths, traffic is reduced and journey times fall for all. Convincing enough drivers to change their behaviour for the benefit of the network can be a difficult problem, but mathematicians such as Frank Kelly at the University of Cambridge have worked on solutions.

One is to change the behaviour of individual drivers through incentives such as variable road pricing. The congestion charge zones in London and Stockholm encourage drivers to avoid paths through the city centres, even though this may be the shortest route for an individual. These kinds of changes are very different to traditional transport network modifications, like building new rail lines or widening motorway lanes, and they require a new kind of mathematical analysis.

> "A stable network can quickly become unstable if just a few components fail - learning to model these effects is essential."

Road pricing schemes need little in the way of civil engineering so can be rapidly implemented, while at the same time require the choice between a large range of options for zone boundaries and driver charges. In order to model the possibilities and find the best solution, mathematicians must develop faster and better network algorithms that can compare a larger number of options in a shorter amount of time.

Another solution is to provide people with new information that enables them to make more informed decisions. While working as Chief Scientific Adviser to the Department for Transport, Kelly assisted with the creation of a travel-time map of Cambridge in which journey times on public transport are represented by different colours. Someone moving to the city could use the map to quickly find a home that allows an easy commute to their workplace; information which is difficult to find by using a map alone. This work is now being developed into Mapumental, a user-friendly website created by mySociety, the not-for-profit company behind websites such as TheyWorkForYou.

This past research is already being put to work, but Kelly believes the next mathematical challenge is to closely model the links between different types of networks. He envisages a scenario where a problem in the electrical grid could lead to signal failures on the train networks, causing the administrator of a communications network to be late for work, which in turn creates new problems in the transport network, all because of the networks' interdependencies.

Institute of mathematics & its applications

EPSRC Pioneering research and skills

These kinds of unexpected consequences can even occur in a single network. For example, we know how all of the individual pieces in our communication networks operate – after all, we designed them – but we still lack a firm understanding of how large networks function as a whole. Mathematicians have discovered that changes in individual components can cause large-scale networks to undergo phase transitions, transforming rapidly from one behaviour to another. This means that a stable network can quickly become unstable if just a few components fail, so learning to model these effects is essential.

These models don't yet exist because the early pioneers of the internet designed their networks partly through trial and error, applying techniques that worked and rejecting those that didn't. It was easy for them to experiment with new network software because only a few people would be affected if it failed, but now that the internet is such an essential part of society, this kind of risky modification is no longer acceptable. Instead, mathematicians can model the changes created by new software and observe their potential effects before making them in the real world.

One such modification might help different types of networks work together. Modern smartphones can connect to the internet through both the 3G mobile telephony network and local Wi-Fi hotspots, but this connection can often be unreliable, especially when browsing on the move, because only one connection is active at a time.

The next generation of smartphones could allow simultaneous connections over both 3G and Wi-Fi, so that as one signal fades out another fades in to provide a continuous connection, but to do this mathematicians must design an algorithm that switches signals at the correct speed. Change too slowly and data packets will be lost, but change too quickly and packets will bounce between the two connections, creating problems when the data is reconstructed.

Whether in communications or transport, mathematicians have already made great contributions to all of our valuable networks, but there is still much work to be done. The more we learn about how these networks operate on a grand scale, the more useful, efficient and reliable they become, and continued mathematical research promises to keep our networks running smoothly.

TECHNICAL SUPPLEMENT

Travel-time maps

The travel-time maps created by mySociety software developer Chris Lightfoot offer a new way of looking at our transport networks. They allow users to make more informed decisions about their journeys by highlighting the relationship between travel distance and travel time to help identify the best routes.

The maps are produced by fixing a destination point and arrival time, then scraping the journey planner website TransportDirect for travel-time data between all of the transport interchanges in the region. The software then overlays the map with a grid of points and searches for all transport interchanges within a 15 minute walk from each point. Choosing the interchange that offers the shortest journey time to the chosen destination results in a grid of points at which we know either the journey time or that no journey is possible, and these points can be coloured according to the time.

Not every point on the map will have a defined travel-time however, so producing a smooth map requires a further step. The software extrapolates smooth contours using a solution to Laplace's equation, a differential equation used in many areas of physics such as electromagnetism and fluid dynamics, producing a map that is both useful and aesthetically pleasing.

Network phase transitions

Mathematical physicists like James Clerk Maxwell discovered how statistical changes in the distribution of microscopic gas molecules relate to macroscopic changes such as pressure and temperature, explaining the long-observed phase transition of water into steam. Mathematicians conducting research into network phase transitions face the reverse challenge, attempting to determine unknown phase transitions from the collective behaviour of individual network components, but the techniques they use resemble the work of Maxwell and his contemporaries.

References

Kelly, F. (2008) The mathematics of traffic in networks. In: Gowers, T., Barrow-Green, J., and Leader, I. (eds.) *The Princeton Companion to Mathematics*. Princeton University Press. Pp 862-870. URL: http://www.statslab.cam.ac.uk/~frank/PAPERS/PRINCETON/pcm0052.pdf

Lightfoot, C. & Kelly, F. (2006) Travel-time maps – transforming our view of transport. URL: http://www.statslab.cam.ac.uk/~frank/PAPERS/traveltime.html

Wardrop, J. (1952) Some theoretical aspects of road traffic research. *Proceedings of the Institution of Civil Engineers, Part II* , 1 (3) 325-362. DOI: 10.1680/ipeds.1952.11259

EPSRC Grants

Reference: GR/S86266/01
Title: Mathematical modelling of end to end congestion control

Reference: GR/M09551/01
Title: Mathematical modelling of communication network

The IMA would like to thank Professor Frank Kelly, University of Cambridge for his help in the preparation of this document