MATHEMATICS MATTERS

Predicting Climate Change

Our understanding of climate change draws on expertise from a variety of scientific disciplines, but climate models ultimately rely on advanced mathematical equations. Even the fastest computers in the world can struggle to solve these equations, so we need to deploy new mathematical techniques in the fight against global warming.

Increasing global temperatures, rising sea levels and the disruption of fragile ecosystems: climate change is one of the greatest challenges humanity has ever faced, and could potentially affect billions of lives in the coming century. Scientists around the world are working to tackle the problem with detailed models of our changing climate, and mathematicians are at the heart of these models, solving the difficult equations that no one else can. Researchers in meteorology, physics, geography and a host of other fields all contribute their expertise, but mathematics is the unifying language that enables this diverse group of people to implement their ideas in climate models.

At the centre of all climate models are the Navier-Stokes equations, which describe the movement of liquids and gases such as the atmosphere and ocean. The secrets of the climate system are locked away in these equations, which were first written down in the 19th century, but they are too complex to be solved directly. Instead, climate modellers use computers to find approximate solutions while maintaining a high degree of accuracy. Translating the Navier-Stokes equations into a form that computers can understand is a specialised task, undertaken by mathematicians such as Paul Williams at the University of Reading.

One fundamental problem is the difference between time in the real world and its digital equivalent; unlike the continuously flowing time that we experience, computers can only work in snapshots. Thankfully, the particular form of the Navier-Stokes equations means that knowing the state of the climate at a certain point in time also tells you how quickly the climate is changing. Mathematicians can exploit this to solve the equations one small step at a time, jumping between snapshots in a process known as time-stepping. Williams' research aims to improve this method, making it more accurate without slowing down the calculations.

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These kinds of efficiencies are extremely important when dealing with a system as large as the Earth. Climate models use a three-dimensional grid that covers the entire surface of the planet and reaches high into the atmosphere and deep into the ocean. A typical grid size is 100 km horizontally, meaning that there are only a handful of grid boxes across the entire UK. At this size many of the fine atmospheric and oceanic details are lost, but the grid boxes can't be made any smaller because of the immense computing power this would require. As such, any low-cost increase in accuracy is essential.

Ideally, climate modellers would use a grid size no larger than one metre, but the technology to do this doesn't exist and won't for some time - although computing power is always improving as processors become faster, reaching this level of detail would take centuries. Even just halving the grid size makes the calculations involved over ten times harder, so a computer that is ten times faster won't necessarily run existing climate models ten times quicker. An alternative is to find more efficient mathematical methods like those being developed by Williams.

Even with these methods, there is still the problem of uncertainty in calculations. There are a variety of time-stepping methods, each with their own strengths and weaknesses, but because climate modelling is so complex, the different methods don't always produce results that agree. Determining which method to use is difficult, but this can actually be beneficial as it allows climate scientists to test one method against another and to look for areas of agreement.

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There are dozens of independent climate models used around the world, and aggregating their results gives us a much better picture of our climate than just using a single model. For example, if half of the models predicted average rainfall over the UK will increase in 2100, but the other half said that rainfall will decrease, climate scientists would have a low confidence in that prediction and attempt to refine their models. On the other hand, because all models agree that the Earth's average surface temperature will increase as levels of carbon dioxide rise, we can be confident that reducing emissions will help halt climate change.

Comparing models in this way is like repeatedly flipping a coin to determine whether it is weighted on one side. One flip doesn't give you very much information, but 100 flips allows you to make statistical predictions. If half of those flips land on heads and the other half on tails then the coin is probably fine, but if all 100 land on heads it is reasonable to assume that the coin is weighted and will land on hands every time. This diversity of climate models is also an essential reason for conducting research in the UK: more countries performing their own climate analysis means the entire world benefits from increased certainty. Certainty can only be achieved, however, if the fundamental mathematics behind the models is well understood. Although the Navier-Stokes equations are widely and successfully used in many areas of science and industry, mathematicians aren't yet able to fully explain how they work. The problem is considered so important that the Clay Mathematics Institute have offered a $1 million prize for a proof that furthers our understanding.

Other aspects of the climate aren't even captured by the Navier-Stokes equations, and some atmospheric phenomena lack fundamental mathematical theory behind them. Clouds are the leading source of uncertainty in climate modelling, because meteorologists aren't sure what the equations which describe them should look like. Clouds are also problematic because they occupy a scale much smaller than the 100 km grids currently in use, so the full details of their behaviour are lost.

These unanswered questions show that while current climate models have served us

well, demonstrating that increased carbon dioxide levels lead to a rise in temperature, we must still gain a deeper understanding of all the mechanisms within our atmosphere and ocean if we are to effectively fight global warming By working with climate scientists to create the new and improved climate models, mathematicians can apply their expertise to the difficult equations involved, and help save the planet for future generations.

TECHNICAL SUPPLEMENT

Navier-Stokes equations

The Navier-Stokes equations were first derived by the French physicist Claude-Louis Navier in 1822, but later developed independently by the British mathematician George Stokes in 1845, who wrote the equations in the form still used today. The equations are derived from Newton's second law of motion, force equals mass times acceleration, and describe the relationship between the velocity, pressure, viscosity and density of a moving fluid.

As a linked set of four nonlinear partial differential equations the Navier-Stokes equations are impossible to solve analytically in all but a few trivial cases, hence the need for numerical approximation methods such as those developed by Paul Williams. These methods allow us to apply the Navier-Stokes equations to a range of practical situations, but we don't actually know whether these solutions will always be valid. Until someone proves this to be the case and claims the Clay Mathematics Institute prize, the full mathematical properties of the equations at the heart of climate models will remain a source of mystery.

Time-stepping methods

Climate scientists use many different varieties of time-stepping methods to power their models, and the choice of method can influence the resulting predictions. The most widely used is the "leapfrog" method, so-called because the function and its derivative get from the previous time to the future time by "leaping" over the current time.

The method's success is due to its ease of use and low computational complexity, but its jumping nature can lead to discrepancies between even and odd steps. This can be solved by using a method called the Robert-Asselin filter to smooth the discontinuities, but at the cost of a loss in accuracy. Williams' research modifies the filter in a way that counteracts this loss, producing better models with no noticeable reduction in calculation speed.

References

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