MATHEMATICS MATTERS

Scans on the Brain

Brain scans play a vital role in the treatment of many serious medical conditions, but decoding the signals inside our minds would not be possible without a variety of mathematical techniques.

Medical imaging techniques let us peer inside the human mind, furthering our understanding of how the brain works and offering the possibility of combating devastating conditions such as Alzheimer's disease and schizophrenia. Mathematical research is at the heart of these techniques, working together with technological advancement to create enormous benefits for society.

Neural activity produces small electrical currents inside the brain, and doctors can now monitor these currents in real-time without performing invasive surgery, allowing them to map the role of different brain areas in everyday activities, or watch the effects of new drugs in action. This electrical activity cannot be observed directly, but there are a range of techniques to convert signals measured from outside of the brain into useful information about what is happening inside, and all rely on mathematics in some way

One such technique is magnetoencephalography (MEG). The small electrical currents inside the brain generate a weak magnetic field that extends outside of the skull. By monitoring the variations in this magnetic field we can see how the electric current is changing over time, and thus gain information about the patient's brain activity. Working out the electric current from the magnetic field is an example of what mathematicians call an "inverse problem", and solving it is key to performing MEG scans.

> "A range of modern brain scanning techniques all rely on mathematics in some way."

Mathematicians study inverse problems in a wide variety of contexts besides brain imaging, such as radar detection, weather prediction, and facial recognition. Solving any of these inverse problems is like knowing the answer but not the question - imagine watching a ball fly through the air, then trying to work out who threw it by tracing backwards. With enough information about its speed and position, it is possible to calculate where it came from and how hard it was thrown.

When it comes to MEG, the calculations are more complicated. Different patterns of electrical current can produce the same magnetic field, making it seemingly impossible to work backwards and pick the correct current that matches the patient's actual brain activity. Without the ability to distinguish between electrical currents MEG would be unusable, but Thanasis Fokas of the University of Cambridge and colleagues have shown that even if two magnetic fields look identical, it is possible to uniquely identify certain parts of the electric currents and recreate some of the original brain activity. In the future it might even be possible to find the entire electric current using additional measurements from another type of scanning technique called electroencephalography (EEG) - this is an area of ongoing mathematical research.

Two other imaging techniques work differently from MEG, but mathematics is still crucial to both. Doctors inject the patient with a tracer chemical that is absorbed by the brain cells, with cells that are more active absorbing a larger amount. The chemical is weakly radioactive, emitting particles that can be detected outside of the skull, and using these external measurements to determine brain activity is another inverse problem that can be solved with mathematics.

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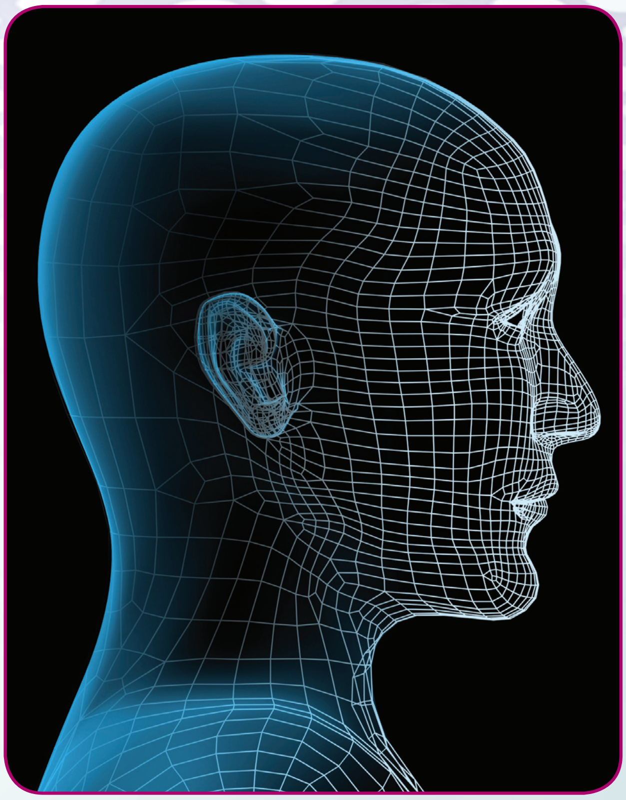
One method, single particle emission computerised tomography (SPECT), uses a single detector that measures all of the radiation emitted along a particular straight line traced from one side of the brain to the other. The level of radiation outside the skull depends on both the amount of radiation generated inside the brain, and the extent to which it is weakened while passing through the brain and skull, known as the attenuation. We can calculate the attenuation with another type of scan called computed tomography (CT), but finding the level of radiation produced by the tracer chemical inside the brain, and thus measuring brain activity, is more difficult.

Mathematicians can use an equation called the "attenuated Radon transform" to describe the radiation measured by the detector in terms of a given radioactive tracer concentration, but SPECT requires the opposite equation – we know the radiation strength outside the skull, but not the amount of tracer chemical inside the brain. Figuring out one from the other means inverting the attenuated Radon transform, a problem that was only solved by Fokas and colleagues within the last decade.

Another method, positron emission tomography (PET), differs from SPECT by using a pair of detectors that measure particles emitted from the brain at the same time but in opposite directions. In this case the measured radiation is described by the plain "Radon transform" combined with a mathematical description of the attenuation. Inverting this combination is mathematically easier than for the attenuated Radon transform, but still a difficult and crucial problem.

The influence of these mathematical techniques has spread beyond just neurology and psychiatry, with both SPECT and PET now playing a crucial role in many other areas of medicine. SPECT is used to diagnose heart diseases, while a combination of PET and CT is the main tool for detecting the size and location of many types of cancer tumours. These kinds of non-invasive scans are some of the most significant advances in medical science in recent years, and were only made possible by crucial developments in mathematics.



The work of mathematicians will continue to play a key role in the future of medical science, and help save lives.

TECHNICAL SUPPLEMENT

Inverse problems

Solving the inverse problem of determining internal brain activity from external magnetic or radioactive signals could only have been achieved with advanced mathematics. In the case of MEG, the magnetic field outside of the head can be written as an integral equation involving the neural current inside the brain, but inverting this integral is far from simple. The equation shows that the radial component of the neural current has no effect on the resulting magnetic field, which means that current levels at different points within the brain can produce the same magnetic field.

Thanasis Fokas and colleagues realised that representing the neural current in appropriate coordinates allowed them to identify certain parts of the current that could be uniquely identified by the magnetic field, provided that the current also fulfilled certain requirements. Without the use of sophisticated mathematics, MEG and other brain scanning techniques would simply not be possible.

Data reconstruction

With any type of inverse problem, it is important to know whether it is possible to reconstruct the information you are interested in with only limited data available. Mathematics is crucial here as well, by filtering noisy data and making allowances for any gaps in the measurements. Techniques such as filtered back projection and maximum likelihood expectation maximization are used in PET and CT, while their application to SPECT is a current subject of mathematical research.

References

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