MATHEMATICS
MATTERS

Volcanic ash: air travel under a cloud

The eruption of the Icelandic Eyjafjallajökull volcano in 2010 not only had newsreaders' tongues in a twist but called a halt to air travel across much of Northern Europe. It was imperative to get planes safely back into the skies as swiftly as possible and mathematics was at the heart of the solution.

On 14 April 2010 the Eyjafjallajökull volcano erupted under a sheet of Icelandic glacial ice for the second time in a matter of weeks. The ice shattered the magma into fine glass and ash which climbed into the sky with the rising volcanic plume. Fearful of its effect on aircraft engines, UK airspace was closed the following day, as the ash cloud headed south, marking the beginning of the largest shut-down of air traffic since the Second World War.

In the ensuing six days 95,000 flights were cancelled across Europe, costing airlines an estimated £1.1 billion. The UK Treasury is thought to have missed out on £30m of revenue from lost air passenger duty, with British hotels, restaurants and shops also taking a significant financial hit. It has been estimated London's economy alone was left £100m out of pocket by the end of the flight ban.

Initially, within the area at risk from ash, the ban was absolute, leaving up to one million British passengers marooned abroad. However, as the saga continued there was mounting pressure to remove the blanket ban and get some flights moving again. As a result, aircraft engine manufacturers released details of a maximum concentration of ash that their engines could withstand. The trouble was that ash concentration is hard to measure directly. In stepped meteorologists at the Met Office's Volcanic Ash Advisory Centre (VAAC) who were mathematically modelling the ash cloud.

The first step is knowing how much ash is injected into the atmosphere in the first place. As the plume rises, it expands and cools. Once its temperature reaches the same as the surrounding air it stops rising. There is a mathematical equation that relates the heat of the initial eruption, and hence the quantity of magma ejected, to the eventual height of the plume. Many of the fine ash particles then collide and stick together, becoming heavier and dropping out of the sky. About 95% of the volcanic ash was deposited to the ground. The key was then to work out where the remaining 5% was going.

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To do this the Met Office used their Numerical Atmospheric-dispersion Modelling Environment (NAME) model, first used to track the fallout from the Chernobyl nuclear disaster. The atmosphere is divided up into a grid, with sections measuring 25km x 25km x 300m. The grid is then populated with real weather data for each area, calculated using a 'Numerical Weather Prediction' model - the Met Office's 'Unified Model' - which in turn uses direct measurements and satellite observations. The data include information on things like temperature and humidity. Next the ash has to be added in. The total mass of ash is divided up into chunks and each chunk is represented on the grid by a particle. The model can then be run to see what happens to each particle, allowing the change in ash concentration to be monitored.

The movement of the ash cloud depends on three main factors: the weather, sedimentation and small-scale turbulence - the NAME model accounts for all three. When it comes to the weather, a set of equations - called the Navier-Stokes equations - run the show. They describe the movement of air in the atmosphere in response to changes in factors such as temperature and pressure. By using the Navier-Stokes equations, meteorologists can work out how the prevailing weather conditions will evolve over the next few days. The predicted winds are then used

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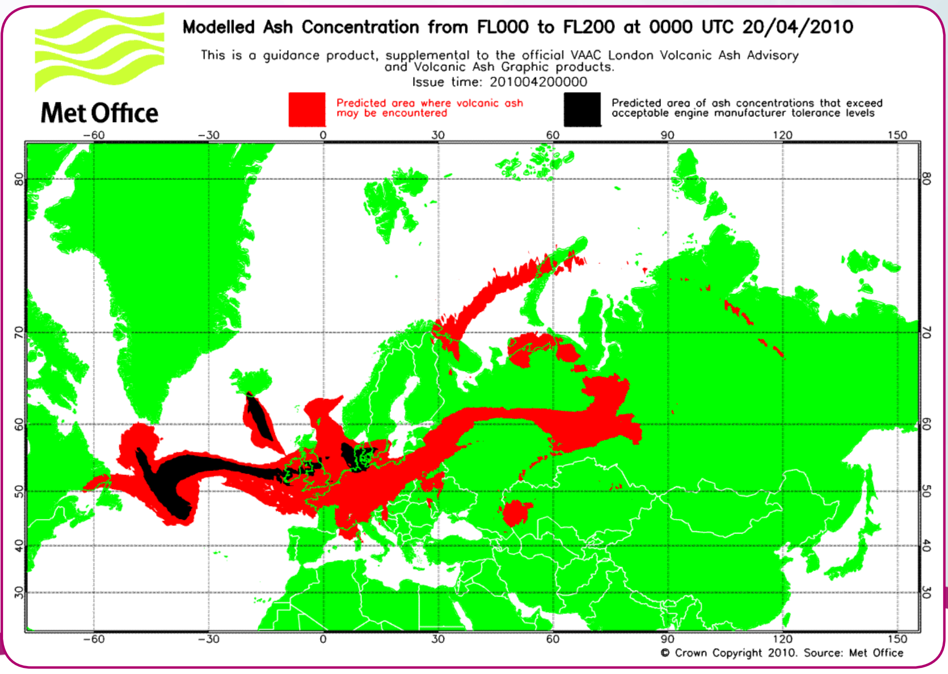
by NAME to move the ash cloud particles from grid area to grid area over time. Gravity also affects the ash, causing some of it to drop to lower altitudes – a process called sedimentation. This change in ash concentration also forms part of the model.

However, despite carving up the atmosphere into relatively small grids, solving the Navier-Stokes equations and factoring in sedimentation, movement of ash on the smallest scales still needs to be taken into account. This small-scale diffusion is caused by eddies – small whirlpools of air that may or not follow the direction of the overall weather pattern. In order to mimic the effect of these eddies, a small random element of motion was applied to each particle.

Once the model had been run, meteorologists at VAAC had a map that predicted how much ash was present over different areas at a particular times. The airspace around the UK was divided up into a new grid, this time measuring 40km x 40km x 20,000 ft, which was then overlaid on the map of ash concentration. At any given time the average concentration of ash in each box could be compared to the engine-safe levels given by the manufacturers, and then assigned “fly” or “don’t fly” status.

In truth, one extra step was required. Just because the average concentration within a box fell within engine safety requirements, didn’t mean there weren’t pockets of ash within the box with a higher concentration. In order to protect against such high density pockets, the average concentration for each box was multiplied by a factor of twenty, a number based on observational evidence for such high density pockets, before being compared with the stated safe concentrations.

On the back of this work, UK airspace was finally re-opened on 20 April allowing a British Airways flight from Vancouver to touch down on the tarmac at Heathrow, the first flight to do so in six full days. Amid the disruption, which had left a million Brits stranded abroad and cost the UK economy billions of pounds, it was mathematics that played a crucial role in returning aircraft to the skies.


Modelled Ash Concentration from FL000 to FL200 at 0000 UTC 20/04/2010

This is a guidance product, supplemental to the official VAAC London Volcanic Ash Advisory and Volcanic Ash Graphic products.
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TECHNICAL SUPPLEMENT

Plume height

The plume height is proportional to the heat of the eruption to the power of about one quarter. The initial placing of the ash on the grids was done under the assumption that the plume rises straight upwards.

Navier-Stokes equations

The Navier-Stokes equations are a set of non-linear partial differential equations derived from Newton’s Second Law of Motion – the one that relates force to changes in motion. The atmosphere in each initial grid is assigned three components of velocity based on current weather and satellite observation parameters.

The equations then resolve the flow of the atmosphere, suggesting how the weather might unfold over a small amount of time – called a time-step. Those new conditions are then put back into the equations for another time-step, and so on, until an iterative picture builds of how the weather, and in particular the winds, will evolve. The winds calculated from the Navier-Stokes equations are then used to calculate the movement of the ash.

Sedimentation

Sedimentation is modelled assuming each actual particle of ash (as opposed to the model particles on the grid, which represent chunks of ash) is a sphere with a fixed density of 2,300 kg m-3. They are then assumed to fall at their terminal velocity – the speed at which the force due to the Earth’s gravity exactly balances the drag of the atmosphere. These sedimentation velocity components are added to each of the three weather-induced velocity components, calculated from the Navier-Stokes equations, to give a fuller picture of ash movement over each time-step.

Small-scale turbulence

The last modelled component of the ash cloud’s velocity is caused by small-scale turbulence – eddies. The effect of eddies is represented by using a type of Markov process known as a diffusion process to model the evolution of the position of the particles. This is equivalent to solving an advection-diffusion equation for the ash concentration field. More sophisticated models where a diffusion process is used for the velocity (instead of position) of the particles are available as options within NAME but are unnecessary for long-distance dispersion problems.

References

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