What is plagiarism in the context of expository work?

Plagiarism can take several forms:

1) Directly copying any part of a text without citation.
2) Paraphrasing any part of a text without citation.
3) Paraphrasing a large part of a text, even with citation. (As merely collating several sections of work from other sources is not original work).
4) Presenting the gist or ideas of a work without citation.

What is not plagiarism?

1) Making short direct quotes of another work, appropriately formatted and cited directly afterwards in the text with a page number.
2) Paraphrasing a short passage, e.g., a couple of sentences, and citing it directly afterwards in the text with a page number.
3) Presenting the gist or ideas of a work with appropriate acknowledgements, in your own style and words. An appropriate acknowledgement phrase is, for example:
   "The material in this section is based on the material from section 5 of [3]."

How can I avoid plagiarizing?

1) Use more than one source.
2) Read the source material and let it sink in for 2-3 days before you start writing so you forget the way it was presented, and only remember the ideas.
3) Ask yourself: what are the main ideas in the material? Don't just look at the headings of the source material for the answer. Organize the ideas for yourself. Put your own mark on them.
4) Consider how you would explain the material to someone who had not understood the source material. Can you explain it in a different way?
5) Use substantially different examples. For instance, you might consider working an exercise from one of the sources.
6) Be particularly careful in the introduction to your paper—that is where a large proportion of cases of plagiarism occur.
7) Check to make sure that none of the telltale signs of paraphrasing show up: Organization of your text is the same as that of the source, sentences in your text can be matched up to sentences in the source, examples in your text are the same as or very close to those in the source.
8) Do a complete job of citing sources—if you have looked at it, it must go into the bibliography. If you have summarized the ideas it contains (but not paraphrased it), you should include a sentence to that effect at the beginning of the section where you do this.

9) Cite paraphrased and quoted material immediately afterwards in the text, including a page number in the source.
10) Do not quote or paraphrase more than a couple of sentences at a time.
11) No more than about 10% of your text should be paraphrased or quoted material.
12) Most important: LEARN AND UNDERSTAND THE MATERIAL WELL. The more time you spend really understanding the material, the more you will find yourself organizing it and explaining it in your own way to yourself. Then when you explain it on paper, you will naturally find yourself doing it in your own, unique way.

What is plagiarism in the context of mathematics?

It can be confusing to understand what is meant by plagiarism and paraphrasing in the context of mathematics, since mathematics is built on a precision of language that often does not give many choices for variation. Also, it is sometimes hard to tell what might be considered as general knowledge and what information requires a reference. To try to make this clearer, I have created some examples below.

<u>Paraphrasing and not paraphrasing a text that deals with a rigidly defined mathematical concept</u>

Here is an example of a text that appears in Wikipedia (“Binomial Coefficients,” 31/1/2011) :

> In mathematics, **binomial coefficients** are a family of positive integers that occur as coefficients in the binomial formula. They are indexed by two nonnegative integers; the binomial coefficient indexed by *n* and *k* is usually written $\binom{n}{k}$, and it is the coefficient of the x^k term in the polynomial expansion of the binomial power $(1 + x)^n$. Arranging binomial coefficients into rows for successive values of *n*, and in which *k* ranges from 0 to *n*, gives a triangular array called Pascal's triangle.
>
> This family of numbers also arises in many other areas than algebra, notably in combinatorics. For any set containing *n* elements, the number of distinct *k*-element subsets of it that can be formed (the *k*-combinations of its elements) is given by the binomial coefficient $\binom{n}{k}$. Therefore $\binom{n}{k}$ is often read as "*n* choose *k*".

Here is an example of a paraphrase of the above text:

> The positive integers that occur as coefficients in the binomial formula are called binomial coefficients. Binomial coefficients are given by two numbers, n and k, and are written as $\binom{n}{k}$. This symbol represents the number that appears as the coefficient of the x^k term in the polynomial expansion of $(1+x)^n$. If we arrange all of the binomial coefficients into rows where each row corresponds to a fixed value of n, and where k goes from 0 to n in the nth row, we get a triangle of numbers called Pascal's Triangle. Binomial coefficients occur in various areas of mathematics, especially combinatorics. If a set has n elements, then there are $\binom{n}{k}$ ways to choose a subset of size k. This is why this symbol is often read as "n choose k".

Notice that in this example,

1) The sentences of the paraphrase correspond almost directly to the sentences in the original document, although the wording has been changed.
2) The organization is exactly the same as the original.

This text would still be a form of plagiarism if a reference to the Wikipedia article were made at the end.

Here is an example of different way of explaining the same idea:

> The binomial coefficients are a set of positive integers that describe the coefficients of x in powers of the binomial (x+1). They are denoted by the symbol $\binom{n}{k}$, which is pronounced "n choose k". Specifically, denotes the kth coefficient in the expanded form of $(x+1)^n$, that is, the coefficient of x^k. These coefficients have many applications in combinatorics and probability. Notably, $\binom{n}{k}$ is the number of subsets of size k of a set of size n. Binomial coefficients have many interesting and useful properties, such as fitting into a diagram called Pascal's Triangle, and are related to many other important sets of numbers, such as the Fibonnaci sequence.

Note that although the definition of the binomial coefficients themselves is the same as in the Wikipedia text, as, in fact, it must be, the sentences do not match up, the paragraph is organized differently, and there is an additional piece of information (of course, the Wikipedia article was also much longer and contained more than is quoted above).

Common knowledge and knowledge requiring citation

First of all, historical background is almost never common knowledge. In the continuation of the Wikipedia article on binomial coefficients, two of the three given pieces of historical information are given references. Really, the third piece (about Halayudha) also needs a reference, and this is noted on the linked page about him.

> The notation $\binom{n}{k}$ was introduced by Andreas von Ettingshausen in 1826,[1] although the numbers were already known centuries before that (see Pascal's triangle). The earliest known detailed discussion of binomial coefficients is in a tenth-century commentary, due to Halayudha, on an ancient Hindu classic, Pingala's chandaḥśāstra. In about 1150, the Hindu mathematician Bhaskaracharya gave a very clear exposition of binomial coefficients in his book Lilavati.[2]

Of course, there are some historical pieces of information that are common knowledge, such as that Newton and Leibnitz developed calculus. But when in doubt, put in a reference.

Whether a piece of mathematical knowledge is considered to be common or not depends on the audience for your work. If one purpose of references is to give credit where credit is due, an equal purpose is to give your reader information on where to look for more on your topic. So, if it seems likely that your reader (who, for this assignment is a third year undergraduate in maths who may not have had any given optional module) will not yet be familiar with a certain piece of maths, give a reference where he or she could read more. It is generally better form to refer to a textbook than to Wikipedia or other internet sources for such things.

For example, consider the text on the next page from the Wikipedia article about group representations (accessed 1/2/2011). Assume that you were going to write about group representations for your report. Since your fellow students will all have had abstract algebra and linear algebra, there is no need to cite any reference for the definition of group, homomorphism, invertible matrix, vector space isomorphism, basis, or injective. However, you don't know that all students have had Vector Spaces, so you do need to cite a reference for the concept of a field and a vector space over a field, you need to say what GL(V) is and provide a reference, and you need to explain the idea of a topological vector space. You of course also need to provide references for all of the new ideas of representation theory that you take from this article, such as representation, representation space, continuous representation, kernel of a representation, etc. If you have used the same source for all of the information in a paragraph (but not paraphrased that source!) it is okay to simply give a single reference at the beginning of the paragraph, such as, "the material in this paragraph not otherwise indicated is based on (…..)." It is even better, though, to find a couple or three different sources, base your representation on the understanding you get from looking at all of them, and then include a sentence at the beginning of the paragraph or section that says, “The ideas in this section are drawn from the sources [1], [2], and [3], and further relevant material can be found there.”

A **representation** of a group G on a vector space V over a field K is a group homomorphism from G to $GL(V)$, the general linear group on V. That is, a representation is a map

$$\rho: G \to GL(V)$$

such that

$$\rho(g_1 g_2) = \rho(g_1)\rho(g_2), \qquad \text{for all } g_1, g_2 \in G.$$

Here V is called the **representation space** and the dimension of V is called the **dimension** of the representation. It is common practice to refer to V itself as the representation when the homomorphism is clear from the context.

In the case where V is of finite dimension n it is common to choose a basis for V and identify GL(V) with GL (n, K) the group of n-by-n invertible matrices on the field K.

If G is a topological group and V is a topological vector space, a **continuous representation** of G on V is a representation ϱ such that the application $\Phi : G \times V \to V$ defined by $\Phi(g,v) = \varrho(g).v$ is continuous.

The **kernel** of a representation ϱ of a group G is defined as the normal subgroup of G whose image under ϱ is the identity transformation:

$$\ker \rho = \{g \in G \mid \rho(g) = id\}.$$

A faithful representation is one in which the homomorphism $G \to$ GL(V) is injective; in other words, one whose kernel is the trivial subgroup $\{e\}$ consisting of just the group's identity element.

Given two K vector spaces V and W, two representations

$$\rho_1: G \to GL(V)$$

and

$$\rho_2: G \to GL(W)$$

are said to be **equivalent** or **isomorphic** if there exists a vector space isomorphism

$$\alpha: V \to W$$

so that for all g in G

$$\alpha \circ \rho_1(g) \circ \alpha^{-1} = \rho_2(g).$$

Appropriate citation format can be found on the Learn page. It doesn’t matter which style of citation you choose (Harvard, MLA, etc.), but make sure all of your references follow the same formatting style.